Event Valence and Subjective Probability*

Adam Brandenburger† Paolo Ghirardato‡
Daniele Pennesi‡ Lorenzo Stanca‡

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Abstract

This paper introduces the signed subjective expected utility (SSEU) model in which an individual's willingness-to-bet (WTB) on an event reflects not only the event's subjective likelihood but also its "valence"—a measure of intrinsic attractiveness or aversiveness of the event. As a result, an event's WTB may be greater than 1 or less than 0. Our model directly extends the subjective expected utility model by weakening the Monotonicity axiom. We show that SSEU accounts for behavioral phenomena such as hedging aversion, the conjunction fallacy, coexistence of insurance and gambling, the choice of dominated actions in strategy-proof mechanisms, and the home equity bias puzzle. We go on to show how to extend SSEU to allow for a stake-dependent (and non-additive) WTB, which accommodates experimental evidence of subjects who violate Independence as well as Monotonicity.

Keywords: Signed probabilities, non-monotonicity, indifference substitution, valence, attractive and aversive states

JEL classification: D81, D83.

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†NYU Stern School of Business, NYU Tandon School of Engineering, NYU Shanghai, New York University, 44 West 4th Street, New York, NY 10012, U.S.A. Email: adam.brandenburger@stern.nyu.edu

‡ESOMAS Department and Collegio Carlo Alberto, Università di Torino, Corso Unione Sovietica, 218 Bis 10134, Turin, Italy. Emails: paolo.ghirardato@unito.it, daniele.pennesi@unito.it, lorenzomaria.stanca@unito.it

1 Introduction

In the most common interpretation of objective probability, probabilities are an idealization of empirical frequencies. As a consequence, the probability of any event must lie between 0 and 1. This property was carried over without change to the world of subjective probability by its pioneers, Ramsey (1926), de Finetti (1937), and Savage (1954). In their interpretation, subjective probability is the mathematical representation of a person's willingness-to-bet (WTB). Under standard rationality requirements, it is a (finitely additive) probability measure, assigning to each event a weight between 0 and 1.

In this paper, we re-examine subjective probability for a decision maker (DM) who envisions both events with subjective probability less than 0 and events with subjective probability greater than 1, that is, a DM whose WTBs are represented by signed probabilities. Choice according to such beliefs can violate one of the basic tenets of rational behavior, namely, the Monotonicity axiom of subjective expected utility (SEU), opening the door to expressing a preference for dominated options. Our main conceptual contribution is an explanation for such dominated choices. We suggest that they can arise from the psychological impact that the DM expects from the occurrence of certain events.

The concept of "hedging aversion" provides a useful starting point for understanding our explanation. Consider the decision of a DM who is offered the possibility to hedge the uncertainty about the outcome of a game played by their favorite sports team. Under SEU (in fact, under any model respecting Monotonicity), we would expect the DM to prefer a well-designed bet that yields a positive monetary payoff if their team loses, to offset their disappointment, to a sufficiently small negative monetary payoff in the case of a win, which partially offsets their pleasure in this case. Instead, in two lab-in-field settings, Morewedge et al. (2018) and Kossuth et al. (2020) found that sports fans preferred no bet to one that paid off in the event of a loss, even when there was no clawback in the event of a win (Morewedge et al., 2018). Donkor et al. (2023) found, in a third lab-in-field setting, that fans bet more on wins by their favorite teams as compared with other teams about whose prospects they were neutral.

In our interpretation, a DM's aversion to hedging is not based on a failure to process properly the likelihoods of events, but is explainable via an intrinsic "valence"—to use a term from psychology—on events.[1] Specifically, the DM's WTB on event E, denoted by $\nu(E)$, results from the (additive) combination of their probabilistic belief about E, denoted by $p(E)$, and the valence that they associate to the event's occurrence, denoted by $\gamma(E)$. Formally, we write $\nu = p + \gamma$, where p is an ordinary (non-negative) probability measure and γ is a second additive set function

[1] There is a large literature in psychology showing that many individuals overweight positive events and underweight negative events (e.g., Weinstein, 1980; Babad and Katz, 1991).

satisfying $\gamma(S) = 0$ and possibly taking negative as well as positive values.[2] Under conventional SEU, a DM's belief about an event and their WTB on it coincide, so that the valence is identically 0. By contrast, our DM attributes a valence to events, with some events viewed as attractive (i.e., $\gamma(E) > 0$) and some other events viewed as aversive (i.e, $\gamma(E) < 0$).[3] We remark that Ramsey (1926) observed the possibility that, for a DM, events might be linked to an intrinsic desire (or the opposite) when he wrote:

> "[T]he propositions ... which are used as conditions in the options offered may be such that their truth or falsity is an object of desire to the subject. This will be found to complicate the problem, and we have to assume that there are propositions for which this is not the case, which we shall call ethically neutral."

In Ramsey's phrasing, our set-up allows for propositions (events) that are not ethically neutral, but, instead, may carry an intrinsic positive (attractive) or negative (aversive) valence. The valence function is the measure of the DM's "ethical non-neutrality."

Going back to the sports betting example, if the DM assigns a negative WTB to the event that their favorite team will lose, then they will forego a hedge—even one that exhibits only upside. As mentioned, such behavior is inconsistent with SEU, and more generally with any model satisfying the well-known Monotonicity axiom formulated by Schmeidler (1989) in his extension of the SEU axiomatization of Anscombe and Aumann (1963). This axiom states that if an act f yields weakly more desirable outcomes than another act g, across all possible states of the world, then f should be weakly preferred to g. In our example, let g be the bet that pays off \$0 regardless of whether the DM's favorite team wins or loses, and f the bet that pays off \$20 if their favorite team loses and \$0 otherwise. Then, under Monotonicity, the DM will strictly prefer f to g, or, at the least, be indifferent. Yet, in the referenced studies, fans displayed a clear preference for avoiding the hedge (i.e., for bet g over bet f).

We replace Monotonicity with a weaker axiom taken from Grant and Polak (2013), called Indifference Substitution, which requires only that if acts f and g yield equally desirable outcomes across all possible states of the world, then f and g should be indifferent. Our representation result shows that replacing Monotonicity by Indifference Substitution in an otherwise standard SEU model yields an additively separable representation of preferences via a Bernoulli utility u on outcomes and a signed probability measure ν on states. The function ν is an additive set function that assigns measure 1 to the overall state space, but may assign measure greater than 1 or less than 0 to some events. We refer to such a representation as "signed subjective expected utility" (SSEU). Paralleling the standard model, the utility u is unique up to affine transformations, and

[2]Note that this is not the usual Jordan decomposition of a signed measure.

[3]From https://dictionary.apa.org/valence: "Valence [is] the subjective value of an event, object, person, or other entity in the life space of the individual. An entity that attracts the individual has positive valence whereas one that repels has negative valence."

the signed probability measure ν is unique. Thus, while the SSEU representation can be seen as a type of state-dependent SEU, it separates and uniquely identifies (state-independent) utility and WTB (but not the decomposition of the latter, as below).

The mentioned decomposition of the WTB ν into the probability p and the valence γ, with the associated interpretation, is the conceptual contribution of this paper. An obvious question concerns the lack of uniqueness of the decomposition: ν is unique, but it admits infinitely many decompositions (p, γ). As we discuss below, this does not pose a problem for the insights we obtain from our interpretation for applications of the SSEU model. However, we also show that when our model is embedded in a dynamic choice setting, the uniqueness problem is narrowed significantly if we make the standard assumption that preferences are linked by dynamic consistency. Then, the allowable decompositions belong to a set, called $\mathcal{T}(\nu)$, which is symmetric around a natural decomposition (p^*, γ^*), and which we refer to as the "canonical" decomposition. The canonical probability p^* can then be shown to be uniquely pinned down by additional consistency axioms, which require that learning whether the realized event has positive or negative weight does not alter WTB ratios. Finally, the canonical decomposition is fully elicitable from preferences (ex-ante and ex-post), so that it is not a mere normalization but a behaviorally identified object.

The canonical decomposition is also useful for developing comparative statics that quantify departures from SEU. We consider two measures. The first calibrates the departure from SEU using the pointwise absolute value of a signed probability measure ν. However, a larger pointwise absolute value may reflect the prior rather than the valence alone. We therefore introduce a belief-invariant order based on the canonical decomposition (p^*, γ^*) of ν: DM 1 is "uniformly more non-classical" than DM 2 if and only if $p_1^* = p_2^*$, and $|\gamma_1^*(s)| \geq |\gamma_2^*(s)|$ for every state s. This approach isolates pure valence effects from beliefs.

Applications. We apply the SSEU model with our additive decomposition interpretation to provide a unified explanation of several well-known behavioral puzzles or anomalies. First, we return to hedging aversion (Morewedge et al., 2018; Kossuth et al., 2020; Donkor et al., 2023) and demonstrate this effect in our model. Second, we look at the conjunction fallacy (Tversky and Kahneman, 1982, 1983) from the point of view of a decomposition $\nu = p + \gamma$. We establish and interpret a sufficient condition on the ordinary probability measure p and the valence γ for the conjunction-fallacy effect to arise. Then, we turn to the classic Friedman and Savage (1948) paradox of the co-existence of insurance and gambling behavior. We show that if a risk-averse DM has a sufficiently positive valence for the event of winning a lottery, they may simultaneously purchase a (fair) insurance contract and the lottery. Our next application is an explanation of empirical evidence that some individuals choose dominated strategies in strategy-proof mechanisms, such as the well-known deferred acceptance algorithm (Hassidim et al., 2016; Dreyfuss et al., 2022; Shorrer and Sóvágó, 2023). The key to our resolution is that the DM has a sufficiently negative

valence for not being matched with their first choice. Finally, we examine the well-known home equity bias puzzle (French and Poterba, 1991; Van Nieuwerburgh and Veldkamp, 2009). Under our SSEU model, investors privilege equities from their home nation when they assign positive valence to the event that returns of their own nation are high, which can be seen as a form of home pride or attachment.

While alternative explanations for each of the above puzzles have been offered (and they will be discussed in the related section), we find it appealing that SSEU and our valence interpretation can be used to provide a unified analysis of all of them. This is true irrespective of the specific decomposition that the DM "uses."

Extensions. These applications notwithstanding, the parsimony of SSEU naturally entails a cost in the form of a reduced fit. For example, we expect our interpretation of WTB as the sum of probabilistic beliefs and (non-zero) valence to be more tenable when the DM is evaluating bets with small monetary stakes. Also, a more complete theory of the "genesis" of a DM's WTB should take into account other established reasons for departures from the SEU benchmark, such as ambiguity attitudes (Ellsberg, 1961). For this reason, in a later section of the paper, we provide a significant extension of our model that allows for stake dependence as well as ambiguity sensitivity in the formation of WTB. This model also accommodates the experimental results in Schneider and Schonger (2019), who observed subjects jointly violating Monotonicity and Independence, the other key axiom of the SEU model.

Structure. The organization of the rest of the paper is as follows. Section 2 lays out our decision framework, presents the SSEU representation, and introduces and discusses the decomposition of WTB into probabilistic beliefs and valence. Section 3 introduces our four axioms—Weak Order, Independence, Archimedean, and Indifference Substitution—and states our representation theorem. Section 4 presents some simple characterization and comparative statics results. Section 5 contains our explanations of hedging aversion, the conjunction-fallacy effect, the paradox of the coexistence of insurance and betting behavior, the puzzle of choice of dominated strategies in strategy-proof mechanisms, and the home equity bias puzzle. Section 6 extends the model by weakening Independence to obtain a representation of preferences that allows for a stake-dependent WTB. Section 7 concludes with a discussion of other papers that relax Monotonicity or employ signed probabilities (e.g., Dekel et al., 2001; Grant and Polak, 2013). It also covers some conceptual matters, including the possible objection that our DM would be vulnerable to a Dutch Book (de Finetti, 1937) and the relationship of our work to complexity aversion (e.g., de Clippel et al., 2024; Puri, 2024; Gu and Chan, 2024). The appendices contain additional results and the proofs.

2 Preliminaries and Basic Model

2.1 Choice Setting

Consider a finite[4] set S of *states of the world* and a set X of *consequences*. A subset $E \subseteq S$ is called an *event*. We denote by $\mathcal{F}$ the set of all functions (called *acts*) $f : S \to X$.

Given any $x \in X$, define $x \in \mathcal{F}$ to be the *constant* act such that $x(s) = x$ for all $s \in S$. With the usual slight abuse of notation, we thus identify X with the subset of the constant acts in $\mathcal{F}$. If $f, g \in \mathcal{F}$, and an event $E \subseteq S$, we denote by $fEg \in \mathcal{F}$ the act that yields $f(s)$ if $s \in E$ and $g(s)$ if $s \notin E$. Given $f \in \mathcal{F}$ and $u : X \to \mathbb{R}$, $u(f)$ denotes the function $s \mapsto u(f(s))$.

We assume, in addition, that X is a convex subset of a vector space. We can then define for every $f, g \in \mathcal{F}$ and $\alpha \in [0, 1]$ the act $\alpha f + (1-\alpha)g \in \mathcal{F}$, which yields $\alpha f(s) + (1-\alpha)g(s) \in X$ for every $s \in S$. Given $u : X \to \mathbb{R}$, we say that it is *affine* if $u(\alpha x + (1-\alpha)y) = \alpha u(x) + (1-\alpha)u(y)$ for any $\alpha \in [0, 1]$ and $x, y \in X$, and *nontrivial* if $u(x) > u(y)$ for some $x, y \in X$. Finally, given two functions $u, u' : X \to \mathbb{R}$, we write $u \approx u'$ if there exist $a, b \in \mathbb{R}$ with $a > 0$ and $u'(x) = au(x) + b$ for every $x \in X$.

2.2 Signed Subjective Expected Utility

We model the decision maker's (DM's) preferences on $\mathcal{F}$ by a binary relation $\succcurlyeq$. As usual, $\succ$ and $\sim$ denote, respectively, the asymmetric and symmetric parts of $\succcurlyeq$. If $f \in \mathcal{F}$, an element $x_f \in X$ is a certainty equivalent for f if $f \sim x_f$.

A signed probability measure is a function $\nu : S \to \mathbb{R}$ such that $\sum_{s \in S} \nu(s) = 1$. Given $E \subseteq S$, set $\nu(E) = \sum_{s \in E} \nu(s)$. The set $\Delta(S)$ denotes the set of all signed probability measures on S, that is

$$\Delta(S) = \left\{ \nu : S \to \mathbb{R} : \sum_{s \in S} \nu(s) = 1 \right\}.$$

We denote by $\Delta^+(S)$ the set of ordinary (non-negative) probability measures on S, i.e.

$$\Delta^+(S) = \left\{ \nu : S \to [0, \infty) : \sum_{s \in S} \nu(s) = 1 \right\}.$$

Given $\phi : S \to \mathbb{R}$ and $\nu \in \Delta(S)$, set

$$\int_S \phi d\nu := \sum_{s \in S} \phi(s)\nu(s).$$

We then have a natural extension of the SEU model to the case in which the WTB is represented by a signed probability measure.

[4] To simplify the analysis, we assume a finite state space throughout. However, with appropriate modifications, most results can be extended to an infinite state space.

Definition 1. We say that (u, ν) is a **signed subjective expected utility** (**SSEU**) representation of $\succcurlyeq$ if there are a nontrivial affine function $u : X \to \mathbb{R}$ and $\nu \in \Delta(S)$ such that

$$f \succcurlyeq g \iff \int_S u(f)d\nu \geq \int_S u(g)d\nu.$$

In applications, the signed probability measure ν is often restricted so that the WTB on each state lies between -1 and 1; see, for example, Brandenburger et al. (2024a). We refer to this special case of SSEU as exhibiting *unit-bounded* beliefs.

Definition 2. We say that $\succcurlyeq$ admits a **unit-bounded SSEU** representation if there exists an SSEU representation (u, ν) of $\succcurlyeq$ such that $\nu(s) \in [-1, 1]$ for all $s \in S$.

2.3 Valence Representations

As discussed in the Introduction, we interpret a signed WTB ν as the result of the additive combination of a (classical) probabilistic belief and a function that quantifies the *valence* – whether aversiveness or attractiveness– that the DM ascribes to an event. Formally:

Definition 3. A function $\gamma : 2^S \to \mathbb{R}$ is called a **valence** if

1. For all $E, F \subseteq S$, if $E \cap F = \emptyset$, then $\gamma(E \cup F) = \gamma(E) + \gamma(F)$;
2. $\gamma(E) + \gamma(E^c) = 0$.

Note that Conditions 1 and 2 imply $\gamma(S) = 0$, $\gamma(\emptyset) = 0$, and $\gamma(A) = \sum_{s \in A} \gamma(s)$. Condition 1 says that the valence of an event depends on the valence of the states comprising the event (we relax this property in Section 6). Condition 2 says that if an event has a positive valence, its complement has a negative valence (again, we relax this assumption in Section 6). Moreover, the valence of the entire state space is 0.

Definition 4. We say that (u, p, γ) is a **valence representation of** $\succcurlyeq$ if there are a nontrivial affine function $u : X \to \mathbb{R}$, $p \in \Delta^+(S)$, and a valence function $\gamma : 2^S \to \mathbb{R}$ such that

$$f \succcurlyeq g \iff \int_S u(f)dp + \int_S u(f)d\gamma \geq \int_S u(g)dp + \int_S u(g)d\gamma.$$

If one defines the set function $\nu = p + \gamma$, the properties of the probability p and of the valence γ ensure that ν is a signed probability measure with $\nu(S) = 1$. Indeed, both p and γ are additive over disjoint unions and $\nu(S) = p(S) + \gamma(S) = 1$, so that $\nu \in \Delta(S)$. Therefore, any valence representation can be rewritten as an SSEU representation, as defined above.

To help in further interpreting a valence representation, consider a bet xEy with $x \succcurlyeq y$. Given a valence representation (u, p, γ) of ν, its SSEU value is

$$V(xEy) = \underbrace{p(E)u(x) + (1 - p(E))u(y)}_{\text{SEU component}} + \underbrace{\gamma(E)(u(x) - u(y))}_{\text{valence component}}.$$

The value of the bet xEy on E is the sum of a classical SEU component and a non-classical valence component. Since $x \succcurlyeq y$, $u(x) \geq u(y)$, so that a positive (resp. negative) valence $\gamma(E)$ will increase (resp. decrease) the overall value of the bet xEy. In particular, if $u(x) = 1$ and $u(y) = 0$, then $V(xEy) = p(E) + \gamma(E)$, highlighting the decomposition of the DM's WTB on E into likelihood and valence components.

It is important to observe that an event commanding a WTB larger than 1 is not necessarily perceived by the DM as a "sure event," i.e., as an event with likelihood equal to 1. The DM can assess $p(E) + \gamma(E) > 1$ while $p(E) < 1$. An analogous remark applies to events that command a negative WTB.

Clearly, any preference with a valence representation (u, p, γ) admits an SSEU representation (with u and $\nu = p + \gamma$). But more is true.

Proposition 1. *The binary relation $\succcurlyeq$ has an SSEU representation if and only if it has a valence representation.*

To prove the converse direction, we obtain a valence representation by decomposing ν as follows. Denote by $\|\nu\|_v$ the total variation of ν (i.e., $\|\nu\|_v = \sum_{s \in S} |\nu(s)|$), and then let

$$p = \frac{|\nu|}{\|\nu\|_v} \equiv p^* \text{ and } \gamma = \nu - p^* \equiv \gamma^*. \tag{1}$$

Indeed, as mentioned before, there are infinitely many possible decompositions (p, γ) of ν. Therefore, based on the sole observation of the DM's choices it is not possible to identify which decomposition the DM could be using to form their WTB. However, we call (p^*, γ^*) the *canonical decomposition* of ν, and the function γ^*, extended to all subsets of S using the properties of a valence, the *canonical valence* of ν. The reason is that (p^*, γ^*) enjoys some specific properties in terms of uniqueness, observability, and dynamic extendibility which justify devoting special attention to it. Section 3.3 below provides the details.

Remark 1. *A preference relation with an SSEU representation (u, ν) can also be given a state-dependent representation. Indeed, define the set of positive states $P \equiv \{s \in S : \nu(s) \geq 0\}$, strictly positive states $P_+ \equiv \{s \in S : \nu(s) > 0\}$, and negative states $N \equiv \{s \in S : \nu(s) < 0\}$.*[5] *Then, the preference $\succcurlyeq$ has a state-dependent representation of the form*

$$V(f) = \sum_{s \in S} p^*(s) U(f(s), s),$$

where

$$U(x, s) = \begin{cases} \|\nu\|_v \, u(x) & \text{if } s \in P, \\ -\|\nu\|_v \, u(x) & \text{if } s \in N. \end{cases}$$

[5] All these sets can be derived from preferences. For instance, it is straightforward to verify that $P_+ = \{s \in S : x\{s\}y \succ y \text{ for some } x \succ y\}$ and $N = \{s \in S : y \succ x\{s\}y \text{ for some } x \succ y\}$.

Indeed, for any s with $\nu(s) \neq 0$, $\nu(s) = \frac{|\nu(s)|}{\|\nu\|_v} \frac{\nu(s)}{|\nu(s)|} \|\nu\|_v = p^*(s) \cdot \text{sign}(\nu(s)) \cdot \|\nu\|_v$. This is a very special type of state dependence.[6] *It involves only two state-dependent utilities, which differ in the sign of the marginal utility, with one positive and the other negative, each of which makes the same relative comparisons between consequences. Indeed, consider two strictly positive states $s, s' \in P_+$. The state-dependent utilities $U(\cdot, s)$ and $U(\cdot, s')$ are cardinally equivalent since they differ only by a positive constant. The same is true for any pair of negative states. Most importantly, our state-dependent utilities are uniquely identifiable from revealed preferences, because u is cardinally unique and ν, and therefore $||\nu||_v$, is unique.*

3 Axiomatic Characterization

3.1 The Basic Axioms

This section introduces the axioms characterizing our SSEU model. Start with the standard Anscombe-Aumann axioms that characterize SEU.[7]

Axiom 1 (Weak Order - WO)**.** $\succcurlyeq$ is complete and transitive. Moreover, there exist $f, g \in \mathcal{F}$ such that $f \succ g$.

Axiom 2 (Independence - I)**.** If $f, g, h \in \mathcal{F}$ and $\gamma \in (0, 1]$, $f \succcurlyeq g$ implies $\gamma f + (1-\gamma)h \succcurlyeq \gamma g + (1-\gamma)h$.

Axiom 3 (Archimedean - A)**.** If $f, g, h \in \mathcal{F}$ and $f \succ g \succ h$, there are $\alpha, \beta \in (0, 1)$ such that $\alpha f + (1-\alpha)h \succ g \succ \beta f + (1-\beta)h$.

Axiom 4 (Monotonicity - M)**.** For every $f, g \in \mathcal{F}$, $f(s) \succcurlyeq g(s)$ for every $s \in S \implies f \succcurlyeq g$.

Recalling the discussion in the Introduction, we weaken Monotonicity as follows[8]

Axiom 5 (Indifference Substitution - IS)**.** For every $f, g \in \mathcal{F}$, $f(s) \sim g(s)$ for every $s \in S \implies f \sim g$.

The IS axiom appears in Grant and Polak (2013), where it is called Substitution. Under IS, two acts that yield equivalent payoffs in all states are deemed indifferent. To illustrate the weaker scope of IS compared with Monotonicity, consider two payoffs $x \succ y$. Under Monotonicity, it is

[6] The identification of probabilistic beliefs for general state-dependent utility preferences requires rich data including hypothetical or conditional preferences (e.g., Karni et al., 1983a; Lu, 2019).

[7] The Independence axiom could be weakened, along the lines of Principi et al. (2023).

[8] One potential explanation for violation of Monotonicity is that this entails a subtle form of "state independence" of preferences, or weak separability, which may be overly restrictive. Monotonicity implies that, if $xEf \succ yEf$ for some event E and some act f, then $xFg \succcurlyeq yFg$ for all events F and acts g. In particular, if $x \succcurlyeq y$ then $xEf \succcurlyeq yEf$ for all events E and all acts f.

necessary that $x \succcurlyeq xEy \succcurlyeq y$ for all events E. However, IS allows for the possibility of $xEy \succ x$, that is, that the event E be "attractive," or that $y \succ xEy$, that is, that E be "aversive."

Replacing M with IS characterizes our SSEU representation.

Theorem 1. *The binary relation $\succcurlyeq$ satisfies axioms WO, I, A, and IS if and only if it has a SSEU representation (u, ν). Moreover, if (u', ν') is another SSEU representation of $\succcurlyeq$, then $u \approx u'$ and $\nu' = \nu$.*

It therefore follows from Proposition 1 that the axioms in Theorem 1 are necessary and sufficient to obtain a valence representation of $\succcurlyeq$. Appendix B further characterizes the special case of SSEU preferences that admit a unit-bounded representation.

The next result shows that, in the context of SSEU preferences, imposing M in place of IS is tantamount to assuming the existence of a valence representation of $\succcurlyeq$ under which all events have zero valence (or, equivalently, assuming that all states are positive).

Corollary 1. *Suppose that a binary relation $\succcurlyeq$ satisfies axioms WO, I, A, and IS. Then the following are equivalent:*

(i) $\succcurlyeq$ *satisfies axiom M,*

(ii) $\gamma^*(E) = 0$ *(equivalently,* $\nu(E) = p^*(E)$*) for all* $E \subseteq S$*,*

(iii) $P = S$*.*

3.2 Some Examples of SSEU Preferences

To provide additional perspective, we present a few examples of SSEU preferences. In the first case, we employ the canonical decomposition. The other three cases employ decompositions that are intuitive but not the canonical ones.

Example 1. Suppose there are only two states s_1, s_2 and ν is such that $\nu(s_1) = 1 + \epsilon$ and $\nu(s_2) = -\epsilon$ for some $\epsilon \geq 0$. In this case, $||\nu||_v = 1 + 2\epsilon$ so that $p^*(s_1) = \frac{1+\epsilon}{1+2\epsilon}$ and $p^*(s_2) = \frac{\epsilon}{1+2\epsilon}$. Moreover, $\gamma(s_1) = \frac{2\epsilon(1+\epsilon)}{1+2\epsilon} = -\gamma(s_2)$. Refer to Figure 1. The value of a bet $x \{s_1\} y$ is

$$V(x \{s_1\} y) = \frac{1+\epsilon}{1+2\epsilon} u(x) + \frac{\epsilon}{1+2\epsilon} u(y) + \frac{2\epsilon(1+\epsilon)}{1+2\epsilon}(u(x) - u(y)).$$

△

Example 2. Suppose that for some $s_1, s_2 \in S$ $\gamma(s_1), \gamma(s_2) \neq 0$ and $\gamma(s) = 0$ otherwise. Then the definition of valence implies $\gamma(s_1) = -\gamma(s_2)$. Assume $\gamma(s_1) > 0$. Then

$$V(f) = \int_S u(f) dp(s) + \gamma(s_1) \left(u(f(s_1)) - u(f(s_2))\right).$$

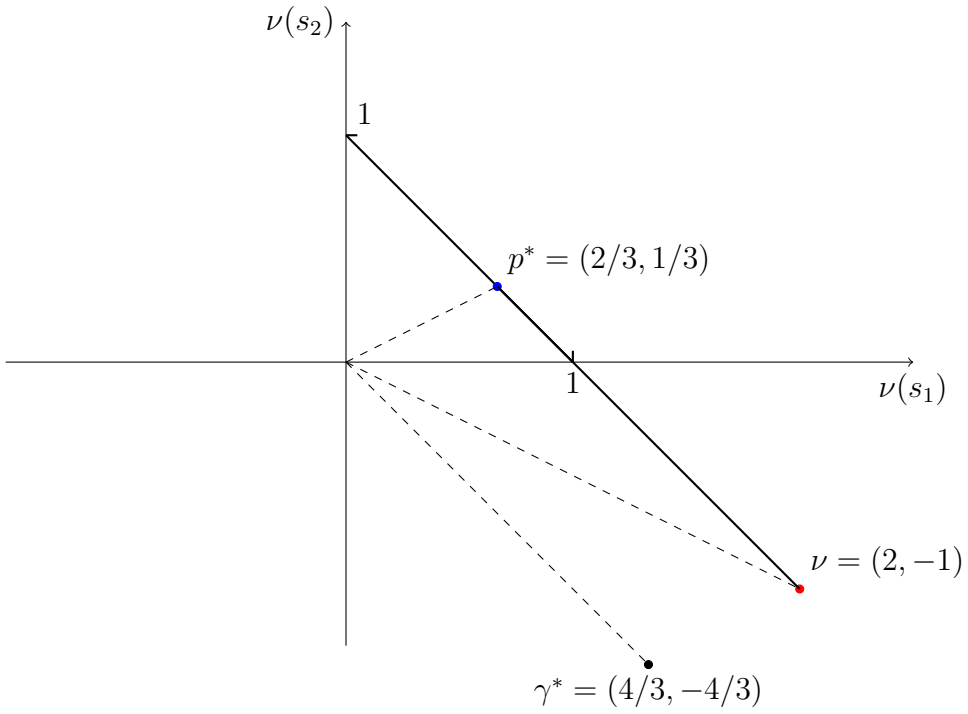


Figure 1: The measure ν and its canonical p^* in the case $\epsilon = 1$.

Consider the bet xEy for some event E with $s_1 \in E$ and $s_2 \in E^c$. Then

$$V(xEy) = p(E)u(x) + (1 - p(E))u(y) + \gamma(s_1)(u(x) - u(y)),$$

and the WTB on E is $\nu(E) = p(E) + \gamma(s_1)$. For general acts, if $f \succcurlyeq g$, then

$$\int_S u(f)dp - \int_S u(g)dp \geq \gamma(s_1)\left[u(g(s_1)) - u(g(s_2)) - u(f(s_1)) + u(f(s_2))\right].$$

Suppose $u(g(s_1)) - u(g(s_2)) - u(f(s_1)) + u(f(s_2)) \geq 0$. Then a preference for f over g implies that the SEU component of the value of f is sufficiently larger than that of g to override the extra value of obtaining relatively more utility in the state s_1 under g, that is, $u(g(s_1)) - u(g(s_2)) \geq u(f(s_1)) - u(f(s_2))$. △

Example 3. Suppose that $S = S_1 \times S_2$ and that $\gamma(S_1 \times E_2) = 0$ for all $E_2 \subseteq S_2$. That is, the states in S_2 are "ethically neutral" and p is a product measure $p(E \times F) = p_1(E)p_2(F)$ for some probability measures p_1, p_2.

Consider the bet xEy with $E = S_1 \times E_2$, then

$$V(xEy) = p_2(E_2)u(x) + (1 - p_2(E_2))u(y)$$

while a bet xFy with $F = F_1 \times S_2$ has value

$$V(xFy) = p_1(F_1)u(x) + (1 - p_1(F_1))u(y) + \gamma(F_1, S_2)(u(x) - u(y)).$$

In this example, there is a source of uncertainty, namely S_1, that triggers a valence response. △

Example 4. In the spirit of the Radon-Nikodym derivative, consider a (non-canonical) valence given by $\gamma(s) = k\phi(s)p(s)$ for some $k \geq 0$ and a distortion function $\phi : S \to \mathbb{R}$ with $\int_S \phi dp = 0$ (in the spirit of the Vector Expected Utility of Siniscalchi, 2009). Then

$$V(f) = \int_S u(f)dp + k \int_S u(f)\phi dp.$$

The WTB on event E is $\nu(E) = p(E) + k \int_E \phi dp$. △

3.3 Decompositions in a Dynamic Environment

As we observed earlier, any preference $\succcurlyeq$ which satisfies the axiomatic requirements listed in Theorem 1 has an SSEU representation with a cardinal u and unique ν. However, the decomposition of ν in the components (p, γ) is far from unique. In this subsection, we show that when our model is put into a dynamic environment, requiring a suitable form of dynamic consistency between conditional preferences goes a long way in reducing the set of allowable decompositions. Specifically, only some decompositions of ν yield posteriors that represent ex post preferences obtained by dynamic consistency, and among these decompositions the one identified by Eq. (1) is a natural focal point. This explains why we call it the *canonical* decomposition of ν.

Assume that we can observe *both* the ex-ante preference of the DM $\succcurlyeq$ and their ex-post preference conditional on the observation of any event $E \subseteq S$, which we denote by$\succcurlyeq_E$ (clearly, we have $\succcurlyeq_S \equiv \succcurlyeq$). We impose a standard requirement of recursivity, namely, consequentialism paired with dynamic consistency (e.g., Hanany and Klibanoff, 2009), adapted to our setting and restricted to events E with all negative or all positive states. Formally, recall the definitions of the sets P (resp. N) of positive (resp. negative) states in Remark 1 (and footnote 5), and let

$$\mathcal{P} \equiv \{ E \subseteq S : \ E \subseteq P, E \text{ non-null}\} \qquad \text{and} \qquad \mathcal{N} \equiv \{ E \subseteq S : \ E \subseteq N, E \neq \emptyset\}.$$

where $E \subseteq S$ is *non-null* if $xEy \not\sim y$ for some $x \succ y$ (equivalently: $\nu(E) \neq 0$). That is, $\mathcal{P}$ (resp. $\mathcal{N}$) denotes the set of positive (resp. negative) events.[9] We require:

Axiom 6 (Recursivity - R)**.** For every $E \in \mathcal{P} \cup \mathcal{N}$ and all $f, g \in \mathcal{F}$:

1. (Consequentialism) If $f(s) = g(s)$ for all $s \in E$, then $f \sim_E g$.

2. (Dynamic consistency) If $f(s) = g(s)$ for all $s \in S \setminus E$, then $f \succcurlyeq g$ if and only if $f \succcurlyeq_E g$.

This axiom says that each positive or negative event E can be treated as a separate decision problem. Conditional on E, only consequences on E matter, and if two acts differ only on E, their overall ranking is determined by their conditional ranking $\succcurlyeq_E$.

[9]Notice that $\mathcal{N} = \emptyset$ iff $N = \emptyset$, which by Corollary 1 obtains if and only if $\succcurlyeq$ is SEU.

It turns out that the only valence representations (u, p, γ) of $\succcurlyeq$ that are compatible with Axiom R are those for which p belongs to the set $\mathcal{T}(\nu)$ defined as follows:

$$\mathcal{T}(\nu) \equiv \left\{ q \in \Delta^+(S) : \; q(s) = \frac{e^{\theta \mathbf{1}_P(s)}}{\sum_{s' \in S} p^*(s') e^{\theta \mathbf{1}_P(s')}} \, p^*(s) \;\; \text{for all } s \in S \text{ and some } \theta \in \mathbb{R} \right\}. \tag{2}$$

Here, $\mathbf{1}_P$ denotes the indicator function of the set P. Each $q \in \mathcal{T}(\nu)$ is an *exponential tilt* of the canonical p^* which reweighs *all* positive states relative to *all* negative states according to the parameter θ (by multiplying $p^*(s)$ times e^θ). The sign of θ captures ex-ante optimism ($\theta > 0$, tilting towards P), pessimism ($\theta < 0$, tilting towards N), and neutrality ($\theta = 0$, yields back p^*). In Appendix D, we provide an information-theoretic interpretation of the family $\mathcal{T}(\nu)$. Briefly, each $q \in \mathcal{T}(\nu)$ is a posterior belief of the canonical prior p^* generated by the coarsest possible information about the state: information that only changes the relative likelihood of P versus N, but leaves the relative likelihoods of states within each set unchanged. The canonical p^* is the only $q \in \mathcal{T}(\nu)$ which also preserves the likelihood ratios based on the ex-ante absolute WTB.

The following result substantiates the claim made above:

Proposition 2. *Let $(\succcurlyeq_E)_{E \subseteq S}$ be a family of binary relations on $\mathcal{F}$ such that $\succcurlyeq$ has an SSEU representation (u, ν). Then $(\succcurlyeq_E)_{E \subseteq S}$ satisfies Axiom R if and only if (u, p^*, γ^*) is a valence representation of $\succcurlyeq$ and for every $E \in \mathcal{P} \cup \mathcal{N}$, $(u, p^*(\cdot \mid E))$ represents $\succcurlyeq_E$ when $E \in \mathcal{P}$, and $(-u, p^*(\cdot \mid E))$ represents $\succcurlyeq_E$ when $E \in \mathcal{N}$. Moreover, if $p \in \Delta^+(S)$ is such that $(u, p, \nu - p)$ is a valence representation of $\succcurlyeq$ and, for every $E \in \mathcal{P} \cup \mathcal{N}$, $\succcurlyeq_E$ admits the SEU representation $(u, p(\cdot \mid E))$ when $E \in \mathcal{P}$ and $(-u, p(\cdot \mid E))$ when $E \in \mathcal{N}$, then $p \in \mathcal{T}(\nu)$.*

In words, ex-post preferences conditional on $E \in \mathcal{P} \cup \mathcal{N}$ are standard SEU obtained by Bayesian conditioning. On $E \in \mathcal{P}$, the DM evaluates according to $(u, p(\cdot \mid E))$, while on $E \in \mathcal{N}$ the same beliefs apply but the utility flips sign to $-u$, reflecting the change in valence. But the only decompositions which are compatible with (SSEU and) Axiom R are those for which the ex-post belief p is an exponential tilt of the canonical prior p^*.

As mentioned, p^* is an "anchor" for all dynamically consistent belief distortions, because it is uniquely determined by (absolute) WTB ratios. That is, for all states s, s' with $\nu(s') \neq 0$

$$\frac{p^*(s)}{p^*(s')} = \frac{|\nu(s)|}{|\nu(s')|}. \tag{3}$$

Indeed, suppose that $p \in \Delta^+(S)$ is a decomposition of ν that also preserves WTB ratios. Then Eq. (3) yields $p(s) = \beta p^*(s)$ for some $\beta > 0$ and all $s \in \operatorname{supp} |\nu|$, and clearly the same is true for all $s \notin \operatorname{supp} |\nu|$ (since then $p^*(s) = p(s) = 0$). It follows that

$$1 = \sum_{s \in S} p(s) = \beta \sum_{s \in S} p^*(s) = \beta,$$

so that $\beta = 1$ and $p = p^*$. In Appendix C, we show that this observation can be used to provide a complete axiomatization of the canonical decomposition by adding to Axiom R the axiomatic counterparts of Eq. (3), which imply that $\theta = 0$.

We conclude by remarking that from the behavioral identification of the canonical p^* and hence γ^*, the behavioral identification of (u and) ν follows immediately. The latter is somewhat less straightforward for our preferences (it could happen that some bets do not have a certainty equivalent), but it can still be achieved by a slight extension of standard methods. Appendix A provides the necessary details, where we also show that elicitation is particularly simple in the unit-bounded case.

4 Comparative Statics

We introduce two measures of *non-classicality*, i.e., the extent to which a decision maker's SSEU preferences depart from the standard SEU benchmark. Both measures are based on choice-theoretic comparative notions of non-classicality.

The first measure is based on pointwise comparisons of the absolute value of the signed belief, together with its set of negative states. A belief is *more non-classical* when it has a larger set of negative states and attributes to each state a larger absolute value.

However, under our interpretation of the signed-belief decomposition, a larger pointwise absolute value may reflect changes in the prior rather than in valence alone. This fact motivates a second measure based on the canonical decomposition. Under this criterion, greater non-classicality corresponds to the same canonical prior, a larger set of negative states, and a pointwise larger absolute value of the canonical valence function. Notably, our choice-theoretic characterization of this measure implies that this second measure is stronger than the former.

4.1 Basic Assumptions

Assume that outcomes are given by simple lotteries over monetary payoffs, that is, $X = \Delta(\mathbb{R})$. As usual, we identify $x \in \mathbb{R}$ with the degenerate lottery that pays x with probability 1.

We consider two DMs with respective SSEU preferences $\succcurlyeq_1$ and $\succcurlyeq_2$. To avoid conflation from different degrees of risk aversion, we assume that the two DMs have the same preferences over payoffs: $\forall\, x, y \in \mathbb{R}$,

$$x \succcurlyeq_1 y \iff x \succcurlyeq_2 y.$$

Moreover, for the rest of this section $\succcurlyeq_1$ and $\succcurlyeq_2$ are assumed to have SSEU representations (u, ν_1) and (u, ν_2), with $u(x) = x$ for every $x \in \mathbb{R}$. This assumption allows us to abstract from differences in risk attitudes and focus on differences in beliefs; it is possible to extend our comparative statics to arbitrary u, but linear utility allows us to keep the exposition simple.

4.2 A Weaker Comparative Notion

The first comparative statics notion is the following:

Definition 5. DM 1 is **more non-classical** than DM 2 if, for every $s \in S$:

$$\text{for } x < 0: \quad 1\{s\}0 \sim_2 x \implies 1\{s\}0 \preccurlyeq_1 x, \tag{4}$$

$$\text{for } x \geq 0: \quad 1\{s\}0 \sim_2 x \implies 1\{s\}0 \succcurlyeq_1 x \text{ or } 1\{s\}0 \preccurlyeq_1 (-x). \tag{5}$$

Condition (4) says that whenever DM 2 requires at least $-x \geq 0$ dollars of compensation to accept the bet on state s, DM 1 requires no less. When DM 2 is willing to pay $x \geq 0$ to accept the bet, condition (5) allows DM 1 either to be even more willing to pay (a stronger attraction to the bet) or, in the opposite direction, to prefer being paid at least x instead.

As claimed earlier, this notion is characterized in the following result, in which N_i denotes the set of negative states for DM i.

Proposition 3. *The following are equivalent:*

(i) DM 1 is more non-classical than DM 2;

(ii) $N_2 \subseteq N_1$ *and* $|\nu_1(s)| \geq |\nu_2(s)|$ *for all* $s \in S$.

Thus, DM 1 is more non-classical in the sense that state-contingent bets elicit larger monetary stakes. The next example illustrates this result.

Example 5. Suppose $S = \{s_1, s_2\}$ and let $\bar{\nu} = (-\frac{1}{2}, \frac{3}{2})$. Then, every signed probability measure with $\nu(s_1) \leq -\frac{1}{2}$ satisfies $|\nu(s)| \geq |\bar{\nu}(s)|$ for every $s \in S$. Next, let $\hat{\nu} = (\frac{1}{3}, \frac{2}{3})$. Apart from $\hat{\nu}$ itself, every signed probability measure such that $\nu(s_1) \leq -\frac{1}{3}$ or $\nu(s_1) \geq \frac{5}{3}$ satisfies $|\nu(s)| \geq |\hat{\nu}(s)|$ for every $s \in S$. Figure 2 illustrates both cases. △

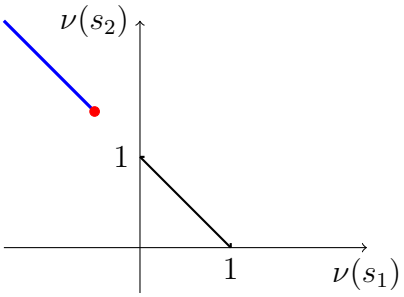
4.3 A Stronger Comparative Notion

As Example 5 shows, Definition 5 is quite permissive in that it conflates changes in beliefs with changes in valence and does not isolate how much non-classicality is driven by valence alone. We therefore introduce a more refined notion.

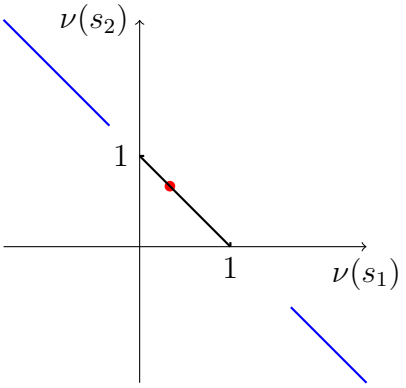
Definition 6. DM 1 is **uniformly more non-classical** than DM 2 if there exists $\alpha \in (0, 1]$ such that, for every $s \in S$:

$$\text{for } x < 0, \quad 1\{s\}0 \sim_2 \alpha x \implies 1\{s\}0 \sim_1 x, \tag{6}$$

$$\text{for } x \geq 0, \quad 1\{s\}0 \sim_2 \alpha x \implies 1\{s\}0 \sim_1 x \text{ or } 1\{s\}0 \sim_1 -x. \tag{7}$$



(a) In red, $\bar{\nu}$. In blue, signed probabilities more non-classical than $\bar{\nu}$. In black, classical probabilities.



(b) In red: $\hat{\nu} = (\frac{1}{3}, \frac{2}{3})$. In blue, signed probabilities more non-classical than $\hat{\nu}$. In black, classical probabilities.

Figure 2: Comparative non-classicality via total variation.

Equation (6) states that whenever DM 2 requires compensation of $-\alpha x \geq 0$ dollars to accept a bet on s, DM 1 requires the higher compensation of $-x \geq 0$ dollars to accept the same bet. Equation (7) states that if DM 2 is willing to pay αx dollars to accept the bet on s, then DM 1 either is willing to pay the higher amount of x dollars to take the bet, or instead requires being paid x dollars to take the same bet.

Definition 6 is stronger than Definition 5. If a decision maker is uniformly more non-classical than another, then they are more non-classical.[10] In addition, the uniform comparative definition requires the two agents to have the same canonical probability and therefore permits fewer comparisons. The following continuation of Example 5 illustrates.

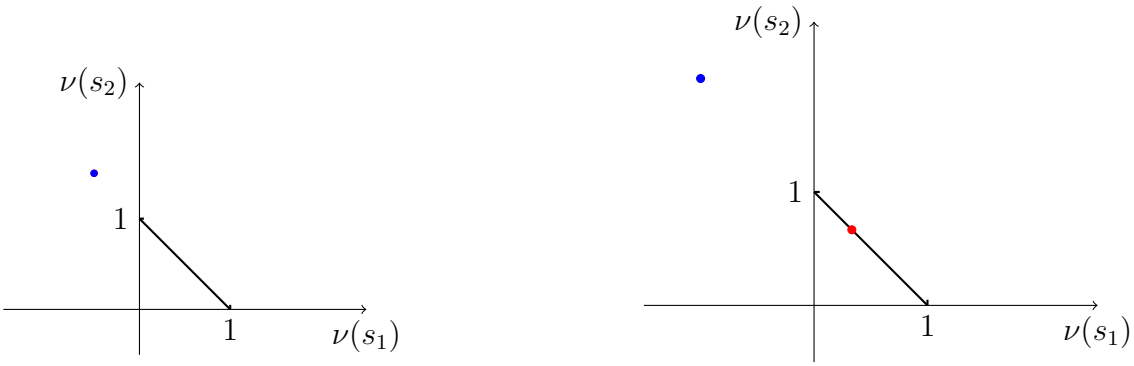
Example 6. For $\bar{\nu} = (-\frac{1}{2}, \frac{3}{2})$, the only signed probability measure uniformly more non-classical than $\bar{\nu}$ is $\bar{\nu}$ itself. For $\hat{\nu} = (\frac{1}{3}, \frac{2}{3})$, the only signed probability measure (other than $\hat{\nu}$) that is uniformly more non-classical is $(-1, 2)$. Figure 3 depicts these cases. △

The next example shows a transformation that makes a signed probability measure uniformly more non-classical.

Example 7. Let ν be a signed probability measure with positive and negative sets P and N. Fix any subset $F \subseteq P$ and assume $\sum_{s \in F} \nu(s) < \frac{1}{2}$, so that

$$c := \frac{1}{1 - 2\sum_{s \in F} \nu(s)} \geq 1$$

[10] Indeed, given $1\{s\}0 \sim_2 x$, one can find $y \in \mathbb{R}$ such that $\alpha y \sim_2 x$. Thus, if $x < 0$, Definition 6 implies that $1\{s\}0 \sim_1 y \preccurlyeq_1 x$. If $x \geq 0$, then $1\{s\}0 \sim_1 y \succcurlyeq_1 x$ or $1\{s\}0 \sim_1 -y \preccurlyeq (-x)$. Therefore, DM 1 is more non-classical than DM 2.



(a) $\bar{\nu} = (-\frac{1}{2}, \frac{3}{2})$: no other point is uniformly more non-classical.

(b) $\hat{\nu} = (\frac{1}{3}, \frac{2}{3})$: the only other uniformly more non-classical point is $(-1, 2)$.

Figure 3: Uniform comparative non-classicality.

is well defined. Define a signed probability measure ν_c by

$$\nu_c(s) = \begin{cases} c\,|\nu(s)|, & s \in P \setminus F, \\ -\,c\,|\nu(s)|, & s \in F \cup N, \\ 0, & |\nu(s)| = 0. \end{cases}$$

Observe that $\sum_{s \in S} \nu_c(s) = 1$ and that $p^*_c = p^*$. The canonical valence of ν_c satisfies

$$\gamma^*_c(s) = \begin{cases} |\nu(s)| \left(c - \dfrac{1}{\|\nu\|_v} \right), & s \in P \setminus F, \\ -\,|\nu(s)| \left(c + \dfrac{1}{\|\nu\|_v} \right), & s \in F \cup N. \end{cases}$$

In words, this transformation concentrates more negative valence on F and correspondingly amplifies the positive valence on the remaining positive states. In Appendix E.6, we show that $|\gamma^*_c(s)| \geq |\gamma^*(s)|$ for every $s \in S$. Therefore, this transformation makes the decision maker with signed belief ν_c uniformly more non-classical than with ν. △

The following result shows that this comparative notion is characterized by a larger set of negative states and point-wise larger absolute canonical valence.

Proposition 4. *The following are equivalent:*

(i) DM 1 is uniformly more non-classical than DM 2;

(ii) $N_2 \subseteq N_1$, $p^*_1 = p^*_2$, *and* $|\gamma^*_1(s)| \geq |\gamma^*_2(s)|$ *for every* $s \in S$.

This result is also useful in presenting a separate tool to identify the canonical probability p^*_1, which coincides with the beliefs p^*_2 of the (SEU) DM 2.

We conclude our discussion by building on Definition 6 to obtain an "absolute" definition of non-classicality. Say that DM 1 is *(uniformly) non-classical* if DM 1 is uniformly more non-classical than DM 2, $\succcurlyeq_1 \neq \succcurlyeq_2$, and $\succcurlyeq_2$ admits a SEU representation. In that case, $N_2 = \emptyset$ and

$\gamma_2^*(s) = 0$ for all $s \in S$. Relative to this benchmark, the characterization in Proposition 4 becomes especially transparent: A decision maker is non-classical whenever $|\gamma_1^*(s)| > 0$ for some $s \in S$.

5 Applications

In this section, we provide some simple examples of how event valence can explain behavior that is hard to reconcile with classical SEU. Unless otherwise noted, we assume throughout that the DM has SSEU preferences $\succcurlyeq$ with some valence representation (u, p, γ). It is important to reiterate that while each of the puzzles we analyze can be explained by some non-expected utility model, an advantage of SSEU is that it provides an explanation of each of them via a simple generalization of SEU.

5.1 Hedging Aversion

Experimental evidence indicates reluctance on the part of sports fans to bet against a win by their favorite team (Morewedge et al., 2018; Kossuth et al., 2020; Donkor et al., 2023). This goes against standard SEU theory, which predicts that a well-designed hedge can mitigate disappointment generated by unfavorable outcomes.

In Morewedge et al. (2018), supporters of a sports team preferred a sure payment of \$0 to a bet paying \$20 if their team lost a certain game. This preference was typically reversed when the bet was on a loss by a team different from the preferred one – even when the two teams had the same likelihood of losing. Rejecting a bet paying \$20 in favor of a sure payment of \$0 is a direct violation of Monotonicity (under the natural assumption that \$20 is preferred to \$0). Also, a difference in WTB on two equally likely events is inconsistent with standard SEU theory, under which the likelihood of an event and the WTB on it coincide.

To see that our SSEU theory can accommodate this pattern of behavior, let E be the event in which the DM's favorite team loses. Then $V(\$0) > V(\$20E\$0)$ whenever

$$u(0) > p(E)u(20) + (1 - p(E))u(0) + \gamma(E)(u(20) - u(0)),$$

which holds if and only if

$$p(E) + \gamma(E) < 0.$$

Intuitively, if the event E has a sufficiently negative valence, the DM's WTB on E becomes negative.

Now let F be the event that a team different from the DM's preferred one loses a game. Again, the DM chooses between a sure payment of \$0 and a bet paying \$20 if this team loses. Even if the two events E and F have the same likelihood, that is, $p(E) = p(F)$, SSEU theory is consistent

with both $V(\$0) > V(\$20E\$0)$ and $V(\$20F\$0) > V(\$0)$ whenever

$$-\gamma(E) > p(E) = p(F) > -\gamma(F).$$

A particular case is $\gamma(F) = 0$, which says that the event F has no valence for the DM. Under SEU, we also have $\gamma(E) = 0$, so that this preference pattern is impossible.[11]

5.2 Conjunction Fallacy

Individuals exhibit the conjunction fallacy when they consider the probability of a conjunction of two events to be higher than the probability of one of the constituent events (Tversky and Kahneman, 1982, 1983). Our SSEU model can accommodate the conjunction fallacy, as the following immediate result illustrates

Fact 1. *For any $E, F \subseteq S$, if $\gamma(E \cap F) - \gamma(F) = -\gamma(F \backslash E) > p(F) - p(E \cap F)$, then $\nu(E \cap F) > \nu(F)$.*

As an example, consider the following "modernization" of a famous experiment by Tversky and Kahneman (1983).[12] The following four events concern a tennis match played by Italian star Jannik Sinner (with their depictions in Table 1):

- E_1 = Sinner wins the match (rounded solid box)
- E_2 = Sinner loses the first set (solid box)
- E_3 = Sinner loses the first set but wins the match (dotted box)
- E_4 = Sinner wins the first set but loses the match (dashed box)

First set \ Match	win	lose
win	s_1	s_4
lose	s_3	s_2

Table 1: Events in the Conjunction Fallacy

[11] Morewedge et al. (2018) find that as the financial gain increases (from \$20), fans are more likely to accept the bet against their team. The generalization of our model in Section 6 can accommodate this preference pattern.

[12] The events in their experiment involved tennis great Björn Borg.

In the Tversky-Kahneman experiment, subjects (on average) ranked event E_1 more probable than event E_3, event E_3 more probable than E_2, and event E_4 as the least probable. A conjunction fallacy arises because event $E_3 = E_1 \cap E_2$, which is the conjunction of events E_1 and E_2. The probability of E_3 should not be larger than that of E_1 or E_2. Tversky and Kahneman explain this phenomenon by suggesting that the different probability judgments may involve different judgment heuristics, thus violating the rules of (classical) probability calculus.

Under our SSEU theory, a DM will exhibit the conjunction fallacy if $\nu(E_3) > \nu(E_2)$. This happens if the valence of the state s_2, where Sinner loses the first set and also the match, is sufficiently negative. For a Sinner fan, satisfaction of this condition could reflect the (anticipated) disappointment after a "total" loss.

Fact 2. *Suppose that $p(s_1) > -\gamma(s_1)$, $p(s_2) < -\gamma(s_2)$, and $p(s_4) + \gamma(s_4)$ is sufficiently small. Then*

$$\nu(E_1) > \nu(E_3) > \nu(E_2) > \nu(E_4).$$

To see this, observe that by the first condition, $\nu(E_1) = p(s_1) + p(s_3) + \gamma(s_1) + \gamma(s_3) > p(s_3) + \gamma(s_3) = \nu(E_3)$. By the second condition, $\nu(E_3) = p(s_3) + \gamma(s_3) > p(s_3) + p(s_2) + \gamma(s_3) + \gamma(s_2) = \nu(E_2)$. Lastly, if $p(s_4) + \gamma(s_4) < \nu(E_2)$, we obtain the desired ranking.

As an example, let $\gamma(s_1) = 0, \gamma(s_2) = -a, \gamma(s_3) = a + b$, and $\gamma(s_4) = -b$, where $a > p(s_2)$ and $p(s_2) + p(s_3) + 2b > p(s_4)$. This valence assignment treats a comeback win as attractive, a straight loss as aversive, and a loss after winning the first set as also aversive. Moreover, $\gamma(E_1) = \gamma(E_3) = a + b$, and the induced signed WTBs satisfy $\nu(E_1) > \nu(E_3) > \nu(E_2) > \nu(E_4)$.

Some recent experimental evidence is consistent with this explanation for conjunction fallacies. In Miao et al. (2025), subjects systematically overvalue conjunctive political–economic events, especially when the outcomes are valence-congruent with their party identity. This yields to conjunction fallacies in both beliefs and choices, which cannot be explained by probability distortions alone, and requires instead state- or valence-dependent preferences. This finding directly relates to our model. If identity-consistent outcomes carry intrinsic value, then the perceived likelihood and attractiveness of events increase with their desirability.

5.3 Coexistence of Insurance and Gambling

Standard SEU with concave or convex utility function struggles to explain why individuals simultaneously buy insurance and lottery tickets. The first purchase reflects risk aversion while the second reflects risk seeking. Friedman and Savage (1948) suggest that risk attitudes can vary with wealth levels, so that insurance and gambling can coexist in sequential purchases. We use the SSEU model to propose an alternative explanation based on valence—one that does not require

sequencing of purchases.[13]

In this section, we assume $X \subseteq \mathbb{R}$. An insurance policy covers a loss $\ell \geq 0$ if event L occurs. The premium is denoted by ψ, and we assume that $0 \leq \nu(L) \leq 1$ and that $\psi = \nu(L)\ell$. A lottery pays $w \geq 0$ if the event E occurs and 0 otherwise. The price π of the lottery ticket satisfies $0 \leq \pi \leq w$. The DM's current wealth is x. Assume that the loss event L and lottery event E are independent under the signed WTB ν.

	L	L^c
f_1	$x - \ell$	x
f_2	$x - \psi$	$x - \psi$

Table 2: Acts f_1 and f_2

The DM compares four options from $\mathcal{F} = \{f_1, f_2, f_3, f_4\}$. The first option f_1 is to buy neither the insurance nor the lottery ticket. The first row of Table 2 shows the associated payoffs. The SSEU is

$$V(f_1) = (p(L) + \gamma(L))u(x - \ell) + (1 - p(L) - \gamma(L))u(x).$$

The second option f_2 is to buy the insurance but not the lottery ticket. The second row of Table 2 shows the associated payoffs. The SSEU is

$$V(f_2) = u(x - \psi).$$

The third option f_3 is to buy the lottery ticket but not the insurance. Table 3 shows the associated payoffs. The SSEU is

$$\begin{aligned} V(f_3) = (p(L) + \gamma(L)) \left[(p(E) + \gamma(E))u(x - \pi + w - \ell) + (1 - p(E) - \gamma(E))u(x - \pi - \ell)\right] + \\ (1 - p(L) - \gamma(L)) \left[(p(E) + \gamma(E))u(x - \pi + w) + (1 - p(E) - \gamma(E))u(x - \pi)\right]. \end{aligned}$$

Finally, with option f_4, the individual purchases both the insurance and the lottery ticket. Table 4 shows the associated payoffs. The SSEU is

$$V(f_4) = (p(E) + \gamma(E))u(x - \pi + w - \psi) + (1 - p(E) - \gamma(E))u(x - \pi - \psi).$$

The coexistence of gambling and insurance requires $f_4 \succcurlyeq f_1$.

Fact 3. *Assume that the utility u is increasing and concave. If winning the lottery has a positive valence such that $p(E) + \gamma(E) \geq 1$, then there is coexistence of gambling and insurance.*

[13] If the price of a lottery ticket is small enough, the sequential explanation requires a large local change in the curvature of the utility function. An alternative sequential explanation of this phenomenon is given by Chateauneuf et al. (2007) with their neo-additive capacities model, which violates SEU.

	L	L^c
E	$x-\pi+w-\ell$	$x-\pi+w$
E^c	$x-\pi-\ell$	$x-\pi$

Table 3: Act f_3

	L	L^c
E	$x-\pi+w-\psi$	$x-\pi+w-\psi$
E^c	$x-\pi-\psi$	$x-\pi-\psi$

Table 4: Act f_4

We see that if utility is concave and winning the lottery is a sufficiently attractive event—that is, if $p(E)+\gamma(E)>1$—an SSEU individual will buy both the insurance policy and the lottery ticket. Note that Fact 3 holds at any given level of wealth x, so that our analysis does not depend on wealth effects. Instead, our mechanism operates by using the DM's valence to offset risk aversion. A sufficiently high valence creates risk-seeking behavior even for a DM with concave utility function u. The condition can be stated in terms of what is known as Jensen's gap (for the probabilistic part p of the valence representation of ν):

$$u[p(E)x+(1-p(E))y]-[p(E)u(x)+(1-p(E))u(y)].$$

If this gap is smaller than the quantity $[u(x)-u(y)]\gamma(E)$, then, despite the concavity of u, the DM will choose the bet between x and y over its p-expectation $p(E)x+(1-p(E))y$.

Fact 4. *For any bet xEy with $x\succcurlyeq y$, if u is increasing and $p(E)+\gamma(E)\geq 1$, then $V(xEy)\geq u(p(E)x+(1-p(E))y)$.*

The condition $p(E)+\gamma(E)\geq 1$ implies $\gamma(E)\geq 0$. Therefore, if u is convex, the above inequality is trivially satisfied.[14] The interesting case is when u is concave, and yet a "risk-averse" DM can prefer a lottery to its expected value.

To link Fact 4 to the coexistence of gambling and insurance, observe that, after the purchase of insurance, buying the lottery induces the binary act $(x-\psi-\pi+w)E(x-\psi-\pi)$. Therefore, by Fact 4, if $p(E)+\gamma(E)\geq 1$, then $V(f_4)=(p(E)+\gamma(E))u(x-\psi-\pi+w)+(1-p(E)-\gamma(E))u(x-\psi-\pi)\geq u\Big(p(E)(x-\psi-\pi+w)+(1-p(E))(x-\psi-\pi)\Big)$. The right-hand side simplifies to $u\Big(x-\psi-\pi+p(E)w\Big)$.

[14] Indeed, $p(E)u(x)+(1-p(E))u(y)+\gamma(E)(u(x)-u(y))\geq u(p(E)(x)+(1-p(E))y)+\gamma(E)(u(x)-u(y))\geq u(p(E)(x)+(1-p(E))y)$.

Hence, we find $V(f_4) \geq u\Big(x - \psi - \pi + p(E)w\Big)$, where $x - \psi - \pi + p(E)w$ is the p-expected wealth from buying the lottery in the presence of insurance.

5.4 Dominated Choice in Strategy-Proof Mechanisms

Empirical evidence suggests that some individuals choose dominated (in the sense of first-order stochastic dominance) strategies in strategy-proof mechanisms, such as the Deferred Acceptance (DA) mechanism much studied in the context of school or college choice (Hassidim et al., 2016; Shorrer and Sóvágó, 2023; Dreyfuss et al., 2022).

Ranking that involve *flipping* or *truncation* are two examples of dominated strategy choices. Under flipping, an individual submits a ranking that reverses the "obvious" order of two alternatives—e.g., ranking a school choice that comes with a scholarship below the same choice without a scholarship. Under truncation, an individual submits a restricted ranking that omits some schools. Both types of behavior are inconsistent with standard SEU, which respects first-order stochastic dominance. However, they are consistent with our SSEU theory.

Consider two schools σ_1 and σ_2. Let $u(x_1) = m_1 > 0$ denote the utility of being matched with school σ_1, $u(x_2) = m_2$ the utility of being matched with school σ_2, and $u(x_0) = 0$ the utility of not being matched. Being matched with both σ_1 and σ_2 is not possible under the DA. There are four possible rankings, which we denote by $\sigma_1 \triangleright \sigma_2$, $\sigma_2 \triangleright \sigma_1$, σ_1, and σ_2. The first two rankings are complete, while the third and fourth are truncations.

Let E_1 be the event of being matched with school σ_1, E_2 the event of being matched with school σ_2, and $E_{\neg 1,2}$ the event of being matched with school σ_2 conditional on not being matched with school σ_1. Submitting the ranking $\sigma_1 \triangleright \sigma_2$ generates the act that yields x_1 if E_1 occurs, x_2 if $E_{\neg 1,2}$ occurs, and x_0 otherwise. An analogous act corresponds to submitting $\sigma_2 \triangleright \sigma_1$. Following Dreyfuss et al. (2022), we assume for simplicity that the probability assessment of being matched to school σ_2 conditional on not being matched to school σ_1 can be written as $p(E_{\neg 1,2}) = (1 - p(E_1))p(E_2)$, and similarly for the event of being matched to σ_1 conditional on not being matched to σ_2.

Submitting a ranking that contains only σ_1 generates the act $x_1 E_1 x_0$ that yields x_1 if E_1 occurs and x_0 otherwise. Likewise, submitting a ranking that contains only σ_2 generates the act $x_2 E_2 x_0$ that yields x_2 if E_2 occurs and x_0 otherwise.

Suppose an individual ranks school σ_1 higher than σ_2, that is, $m_1 > m_2$. Flipping means that the individual submits the ranking $\sigma_2 \triangleright \sigma_1$. The SSEU of submitting a faithful ranking is $\nu(E_1)m_1 + \nu(E_{\neg 1,2})m_2$, whereas the SSEU of the flipped ranking is $\nu(E_2)m_2 + \nu(E_{\neg 2,1})m_1$. Submitting a flipped ranking is preferred if

$$p(E_1)p(E_2)(m_1 - m_2) < (\gamma(E_{\neg 2,1}) - \gamma(E_1))m_1 - (\gamma(E_{\neg 1,2}) - \gamma(E_2))m_2.$$

The left-hand side of this inequality is always positive, and so the right-hand side must be sufficiently positive if flipping is to be optimal. For instance, suppose that all events in the inequality

have valence 0 except for the aversive event $E_{\neg 1,2}$, which corresponds to being matched with the less preferred school σ_2 over the more preferred school σ_1. The inequality then becomes

$$p(E_1)p(E_2)(m_1 - m_2) < -m_2\gamma(E_{\neg 1,2}),$$

which holds if $\gamma(E_{\neg 1,2})$ is sufficiently smaller than 0. By contrast, since $\gamma = 0$ under SEU, this inequality can never be satisfied.

Truncation, by submitting just σ_1 rather than the ranking $\sigma_1 \triangleright \sigma_2$, is preferred if

$$\nu(E_1)m_1 > \nu(E_1)m_1 + \nu(E_{\neg 1,2})m_2,$$

which holds whenever $\gamma(E_{\neg 1,2}) < 0$ is small enough, which in turn holds if there is sufficient aversion to being rejected by the more preferred school.

Our explanation of flipping and truncation behavior in the DA mechanism differs from that in Dreyfuss et al. (2022). These authors suggest that "an applicant who is likely to get matched with a school will feel a loss when matched with any other school (even a better one); this can create attachment to the high-probability school—an endowment effect for schools." From this, they conclude that flipping and even truncation might be observed in highly loss-averse individuals. Our explanation is not dependent on a (high) likelihood of being matched with a particular school, but on the aversiveness of not being matched with a preferred school.

5.5 Home Equity Bias Puzzle

The home equity bias puzzle refers to the phenomenon where investors hold a disproportionately large share of domestic assets in their portfolios, despite the potential benefits of international diversification. French and Poterba (1991) document that U.S. investors allocated over 90 percent of their equity holdings to domestic securities, even though the U.S. market represented less than 50 percent of the global equity market. This behavior contradicts standard portfolio theory, which predicts that risk-averse investors will hold an internationally diversified portfolio to reduce exposure to country-specific shocks.[15]

Even when domestic and foreign stocks have the same distribution of returns, an SSEU investor may care more about states in which the home market does well, or care less (or even dislike) states in which the foreign market does well while the home market does poorly, or both. Thus, SSEU is able to model a "home pride" effect, which generates a tilt toward domestic assets.

[15]The literature has proposed several (non-mutually exclusive) explanations for the home equity bias puzzle, including familiarity and behavioral preferences for domestic assets (Huberman, 2001), information frictions and asymmetric information across borders (Van Nieuwerburgh and Veldkamp, 2009), and differential ambiguity aversion (Epstein and Miao, 2003; Cappelli et al., 2020). See Lewis (1999) and Coeurdacier and Rey (2013) for surveys of the literature.

Let $X = \Delta(\mathbb{R})$ denote the simple lotteries over the reals and let $S_H = \{U, D\}$ be a reduced state space specifying whether the domestic stock market goes up (U) or down (D). Similarly, let $S_F = \{U, D\}$ describe the state of the stock market in a foreign country. Consider finally the global product state space

$$S = S_H \times S_F = \{UU,\, UD,\, DU,\, DD\},$$

where each state $s = (s_H, s_F)$ records Up (U) or Down (D) for the domestic (H) and foreign (F) markets. Denote by $E_H = \{UU, UD\}$ the event that the domestic stock market goes up, and $E_F = \{UU, DU\}$ the corresponding event for the foreign stock market. In each market we assume independent up or down moves with probability $\frac{1}{2}$, so that every state $s \in S$ has $p(s) = \frac{1}{4}$.

Given unitary wealth, we denote by $f_{\text{dom}} = 1E_H0$ the act of investing all wealth in the domestic market, and $f_{\text{for}} = 1E_F0$ the act of investing all in the foreign market. Thus, if the investor invests a share $\alpha \in [0, 1]$ of wealth in the home market, and $1 - \alpha$ in the foreign market, the resulting choice will be $f_\alpha = \alpha\, f_{\text{dom}} + (1 - \alpha)\, f_{\text{for}}$; that is,

$$f_\alpha(s) = \begin{cases} 1, & s = UU, \\ \alpha, & s = UD, \\ 1 - \alpha, & s = DU, \\ 0, & s = DD. \end{cases}$$

Under standard SEU with a strictly increasing and strictly concave utility u, the expected utility is

$$U(f_\alpha) = \sum_{s \in S} p(s)\, u\Big(f_\alpha(s)\Big) = \tfrac{1}{4}\Big[u(1) + u(\alpha) + u(1 - \alpha) + u(0)\Big].$$

For simplicity, we focus on the cases in which the optimal portfolio share is interior. Therefore, the necessary and sufficient first–order condition

$$U'(f_\alpha) = \tfrac{1}{4}\Big[u'(\alpha) - u'(1 - \alpha)\Big] = 0 \quad \Longrightarrow \quad u'(\alpha) = u'(1 - \alpha) \quad \Longrightarrow \quad \alpha = \tfrac{1}{2},$$

yields the unique maximizer

$$\alpha^*_{\text{SEU}} = \tfrac{1}{2},$$

so that SEU prescribes, in this completely symmetric case, equal diversification.

Now consider SSEU preferences with the same probabilistic assessment. Formally, assume that $\nu(E) = p(E) + \gamma(E)$, where γ captures intrinsic attraction or aversion to the realization of returns. Thus, the two markets are probabilistically identical for the DM. However, under SSEU the DM may, contrary to standard theory, intrinsically prefer the home market to outperform the foreign market, so that $\gamma(UD) > \gamma(DU)$.

We show that in this case the DM does not perfectly diversify. To see this, observe that the value of f_α is given by

$$V(f_\alpha) = \sum_{s\in S} \nu(s)\, u\Big(f_\alpha(s)\Big) = \nu(UU)\, u(1) + \nu(UD)\, u(\alpha) + \nu(DU)\, u(1-\alpha) + \nu(DD)\, u(0).$$

We thus obtain the first–order condition

$$V'(f_\alpha) = \nu(UD)\, u'(\alpha) - \nu(DU)\, u'(1-\alpha) = 0 \quad \Longrightarrow \quad \nu(UD)\, u'(\alpha) = \nu(DU)\, u'(1-\alpha).$$

Since u' is strictly decreasing, whenever $\gamma(UD) > \gamma(DU)$, we have $\nu(UD) > \nu(DU)$, and so

$$u'(\alpha) < u'(1-\alpha).$$

Therefore, the SSEU–optimal share satisfies

$$\alpha^*_{\text{SSEU}} > \tfrac{1}{2},$$

and exhibits a bias toward the domestic asset. Thus, as soon as the "domestic-only" state UD carries strictly higher valence weight than its mirror DU, the DM will exhibit a home equity bias.

6 Stake Dependence and Other Generalizations

Given an SSEU preference represented by (u, ν) and a negative state s, a DM will not be willing to buy any bet $x\{s\}y$ regardless of the size of the stakes x and y. This is a consequence of the fact that SSEU preferences satisfy Independence, which yields the separable structure $V(x\{s\}y) = u(x)\nu(s) + u(y)(1-\nu(s))$. While this *stake independence* is a feature that is shared by the classical SEU model and by most of its well-known generalizations,[16] it strikes us as particularly implausible when describing DMs who may attach valence to some states or events. A supporter who rejects a bet on their preferred sports team losing when the payoff is small, might well accept the same bet for a sufficiently large payoff.

In this section, we show how to modify our analysis to allow a DM to have a WTB—and hence valence—that is stake-dependent. The model we develop is very general and the substantial relaxation of Independence makes it possible to incorporate other factors affecting WTB for a DM. For example, we can admit ambiguity attitudes (Ellsberg, 1961). A cost of this additional generality is that the DM's valence function loses its simple additivity structure.

As previewed, stake dependence can appear if Independence is required only in the comparison of constant acts:

[16] Separability in the evaluation of bets on states or events follows from the restriction of the Certainty Independence axiom of Gilboa and Schmeidler (1989) to binary acts. This is the Binary Certainty Independence axiom of Cerreia-Vioglio et al. (2011). In the Savage (1954) setting, it corresponds to the well-known P4 property.

Axiom 7 (Risk Independence - RI)**.** If $x, y, z \in X$ and $\alpha \in (0,1]$, $x \succcurlyeq y$ implies $\alpha x + (1-\alpha)z \succcurlyeq \alpha y + (1-\alpha)z$.

The Archimedean axiom is then replaced by a slightly stronger continuity condition and by the explicit requirement that every act have a certainty equivalent.

Axiom 8 (Continuity - C)**.** For any $f, g, h \in \mathcal{F}$, the sets $\{\alpha \in [0,1] : \alpha f + (1-\alpha)g \succcurlyeq h\}$ and $\{\alpha \in [0,1] : h \succcurlyeq \alpha f + (1-\alpha)g\}$ are closed.

Axiom 9 (Solvability - S)**.** For any $f \in \mathcal{F}$, there exists $x \in X$ such that $f \sim x$.

While very mild, and satisfied by the great majority of all existing models of decision making, Solvability does restrict the way in which a DM can evaluate bets with extremely high or extremely low payoffs. This is not the case for the SSEU model presented earlier, so strictly speaking this model is not a proper generalization of SSEU. (See Remark 2 in Appendix E.10.)

However, this feature is consistent with our objective of trying to encompass a wider range of attitudes towards the valence of an event, and we thus see the following result as a conceptual generalization of SSEU and Theorem 1. Let $B_0(2^S, K)$ be the set of all simple K-valued functions, and say that $I : B_0(2^S, K) \to \mathbb{R}$ is *normalized* if $I(\alpha) = \alpha$ for every $\alpha \in K$, and I is *constant-Archimedean* if $\text{Range}(I) \subseteq K$.

Theorem 2. *The binary relation $\succcurlyeq$ satisfies Axioms WO, RI, C, S and IS if and only if there exists a non-constant, affine function $u : X \to \mathbb{R}$ and a continuous, normalized, and constant-Archimedean functional $I : B_0(2^S, \text{Range}(u)) \to \mathbb{R}$ such that for every $f, g \in \mathcal{F}$, $f \succcurlyeq g$ if and only if $I(u(f)) \geq I(u(g))$.*

We say that a preference $\succcurlyeq$ has a *Bernoullian Continuous* (*BC*) *representation* if there are u and I satisfying the properties listed in Theorem 2. To illustrate the scope of the BC representation, consider any bet xEy with $x \succ y$. The functional I can be written as

$$I(u(xEy)) = \rho_{x,y}(E)u(x) + (1 - \rho_{x,y}(E))u(y),$$

where $\rho_{x,y}(E) = [I(u(xEy)) - u(y)]/[u(x) - u(y)]$. It is easy to prove that, for each $x, y \in X$ with $x \succ y$, the function $\rho_{x,y} : 2^S \to \mathbb{R}$ is a *signed capacity*, that is, a set function satisfying $\rho_{x,y}(S) = 1$ and $\rho_{x,y}(\emptyset) = 0$.[17]

Paralleling our earlier decomposition of a signed probability measure ν, a signed capacity $\rho_{x,y}$ can be decomposed into a capacity and a function reflecting (generalized) valence. That is, we can write

$$\rho_{x,y} = \mu_{x,y} + \Gamma_{x,y},$$

where $\mu_{x,y} : 2^S \to [0,1]$ is a capacity on 2^S and the function $\Gamma_{x,y} : 2^S \to \mathbb{R}$, which we call a *generalized valence*, satisfies (only) $\Gamma_{x,y}(\emptyset) = \Gamma_{x,y}(S) = 0$.

[17] Indeed, if $E = S$, $xSy = x$ and so $I(u(xSy) = u(x)$, implying $\rho_{x,y}(S) = 1$. Similarly if $E = \emptyset$.

Proposition 5. *If the preference $\succcurlyeq$ has a BC representation, for each $x, y \in X$ with $x \succ y$, there is a capacity $\mu_{x,y}$ and a generalized valence $\Gamma_{x,y}$ such that*

$$I(u(xEy)) = \mu_{x,y}(E)u(x) + \Gamma_{x,y}(E)u(x) + (1 - \mu_{x,y}(E) - \Gamma_{x,y}(E))u(y).$$

BC preferences can account for stake-dependent WTB. That is, assuming $X \subseteq \mathbb{R}$, we can accommodate preferences where, for some $x \succ y$,

$$y \succ xEy \quad \text{and} \quad (x+m)Ey \succ y$$

for some $m \in X$. When the stakes are small, the aversiveness of the event E leads the DM to prefer obtaining y for sure over the bet xEy. However, for a sufficiently large payoff $x + m$, the DM reverses their preference. Indeed, assume that $\mu_{x,y}(E) + \Gamma_{x,y}(E) = \rho(E) + \gamma(E)/(|x| + |y|)$ for some capacity ρ and valence function γ (in the sense of Definition 3). If $\gamma(E) < 0$, then, for a sufficiently small x, we can have $u(y) > I(u(xEy))$. However, as x increases to $x + m$, we get $\mu_{x+m,y}(E) + \Gamma_{x+m,y}(E) \approx \rho(E)$, so that $I(u((x + m)Ey)) > u(y)$. Clearly, the issues about the non-uniqueness of the decomposition in SSEU arise *a fortiori* in this less structured setting.

Exploiting the non-additivity of the (signed) capacity $\rho_{x,y}$, the BC model can also reproduce the preference pattern observed experimentally by Schneider and Schonger (2019). They found evidence for violations of the following consequence of Monotonicity: If $xEz \succ yEz$ for some $x, y, z \in X$ and an event E, then $xEz' \succcurlyeq yEz'$ for all $z' \in X$.

Example 8. Consider the special case of the BC model in which the WTB $\rho_{x,y}$ is actually stake-independent, that is, $\rho_{x,y}(E) = \rho(E)$ for all $x, y \in X$ and all $E \subseteq S$. Fix payoffs $x, y \in X$ with $x \succ y$ and consider an event E and a (signed) capacity ρ with $\rho(E) > 0$. Given $z \in X$ with $y \succcurlyeq z$, we have

$$I(u(xEz)) = \rho(E)u(x) + (1 - \rho(E))u(z) > \rho(E)u(y) + (1 - \rho(E))u(z) = I(u(yEz)).$$

That is, $xEz \succ yEz$. Now suppose that $1 - \rho(E^c) < 0$. Then, given $z' \succcurlyeq x$, we have

$$I(u(yEz')) = \rho(E^c)u(z') + (1 - \rho(E^c))u(y) > \rho(E^c)u(z') + (1 - \rho(E^c))u(x) = I(u(xEz')).$$

That is, $yEz' \succ xEz'$. Notice that if the capacity ρ is a signed probability measure, then the conditions $\rho(E) > 0$ and $1 - \rho(E^c) < 0$ cannot be simultaneously satisfied. Also, the capacity ρ is non-convex, since $\rho(E) + \rho(E^c) > 1$. △

7 Discussion and Related Literature

We conclude with some conceptual remarks and further comments on the literature.

a. A Dutch Book Objection There is a possible challenge to our theory, which is that a DM following SSEU might be susceptible to a Dutch Book, and a consequent money pump (de

Finetti, 1937). If the DM's WTB on event E is $\$\nu(E)$, where $\nu(E) > 1$, then a bettor who does not share this assessment will be able to obtain a sure win, by selling the DM a bet with prize \$1 if E obtains and \$0 otherwise—at a price of $\$(\nu(E) - \epsilon)$ where $\epsilon < \nu(E) - 1$. But, while this transaction might be feasible once, it is not clear to us that it will be repeatable. After the first transaction, the DM might well change valence so that $\nu(E) \leq 1$ for subsequent bets. Money pumps can occur only in the presence of unchanging preferences over the course of the pump, and we see no reason to assume that valence might not vary with the DM's betting trajectory. Even for the one-time bet, the sure loss looks troublesome only from an outsider's perspective (assuming the outsider follows classical SEU). From the DM's perspective, the positive valence attached to E implies a (non-monetary) welfare gain from betting on E.

b. Related Work There are other papers in decision theory where signed probabilities arise. Dekel et al. (2001, 2007) study preference over menus of lotteries and provide an axiomatization of an EU representation with respect to a signed probability measure on a subjective state space. In their framework, positive states represent potential future normative preferences, while negative states model possible future "temptations" that could affect preferences. This is a different interpretation from the one we offer here. Perea (2022) considers a setting motivated by game theory and axiomatizes a player's conditional preference relation over acts, where the conditioning is on different signed probability measures ("beliefs") the DM might hold over choices made by other players. Ke and Zhao (2024) present a model of decision making under ambiguity, called cautiously optimistic linear utility, which features a collection of sets of possibly signed (subjective) measures over states. These two papers do not offer an interpretation for the signed probabilities they obtain. Mackenzie (2025) drops Monotonicity from the classic Villegas (1964) axioms and still obtains a unique signed-probability representation of an "inference" ranking of events. Brandenburger et al. (2024a) is another application of signed probabilities—to an examination of the status of the Agreement Theorem (Aumann, 1976) of classical epistemics in a non-classical setting. That work was the motivation for the development of the SSEU model in this paper. In their examination, Brandenburger et al. (2024a) find that conditioning becomes much more complex in a world of signed probabilities, because an event of probability 0 may contain a sub-event of strictly positive or negative probability. In a companion paper to the current one (Brandenburger et al., 2024b), we develop a theory of *signed conditional probability spaces*, extending the classical theory of conditional probability spaces due to Rényi (1955).

Other models relax or omit altogether the Monotonicity axiom. Grant and Polak (2013) relax this axiom in order to represent general mean-dispersion aversion preferences, which include well-known non-monotonic models such as mean-variance utility. In Ellis and Piccione (2017), Monotonicity fails because an individual can misperceive correlations between actions, for instance, by believing that mixing two perfectly correlated assets can reduce risk. De Waegenaere and Wakker (2001) and Bommier (2017) depart from Monotonicity in order to obtain

non-separability of preferences over dates or states of the world, respectively. The SSEU model retains full separability.

Aoyagi et al. (2021) find evidence of violations of monotonicity due to a "coordination fallacy." This fallacy arises in strategic uncertainty settings, where individuals overweight the likelihood of efficient coordination outcomes. Our theory can explain such a fallacy by assigning positive valence to the event of successful coordination.

Our theory is also connected to the literature on state-dependent utility. Under state dependence, the representation typically does not pin down beliefs and utility separately–at least, without additional structure or normalization (e.g., see Karni et al. 1983b; Karni 1985; Schervish et al. 1990; Karni and Mongin 2000; Drèze and Rustichini 2004). Recent work shows how suitable enrichment of observed choice data can restore identification in state-dependent settings (Lu, 2019). By contrast, our assumptions deliver a revealed-preference identification result. Up to the usual positive affine normalization of u, observed choices uniquely determine both the Bernoulli utility u and the measure ν. Moreover, conditional preferences permit unique identification of the canonical decomposition, which is not possible with general state-dependent preferences.

c. Valence and Complexity Recent empirical and theoretical research suggests that aversion to complexity can explain violations of Monotonicity (e.g., de Clippel et al., 2024; Puri, 2024; Gu and Chan, 2024). In cases where the dominant act is perceived as too complex compared with a dominated one, complexity aversion may lead individuals to prefer the dominated option. However, it seems clear that complexity aversion cannot explain aversion to hedging, which involves binary—hence simple—lotteries. Another motivation for relaxing Monotonicity arises from failures in contingent or state-by-state thinking (e.g., Zhang and Levin, 2017; Echenique et al., 2022), which are compatible with the weaker notion of obvious dominance (Li, 2017).[18] Our SSEU preferences display state-by-state thinking and may not respect obvious dominance. For example, if $x \succ y$ then xEy obviously dominates y, but an SSEU DM may find $y \succ xEy$.

d. Anticipatory Utility In the model of Caplin and Leahy (2001), anticipatory emotions influence preferences before uncertainty is resolved, implying that beliefs themselves carry psychological utility. Within our SSEU framework, the valence function can be interpreted as capturing this anticipatory utility. In particular, it accords with the intuition that "(suspense) is positively related (up to a point) to the amount that is at stake on the outcome of an event" (Caplin and Leahy, 2001, p. 73). In evaluating a bet, our valence increases (resp. decreases) the value of the bet as a function of the payoffs.

[18] Obvious dominance states that if, for all $s \in S$, $f(s) \succcurlyeq g(s')$ for all $s' \in S$, then $f \succcurlyeq g$, and $f \succ g$ if the first preference is strict (Zhang and Levin, 2017).

A Appendix: The Elicitation of ν and p^*

Our theory suggests a natural procedure to elicit an SSEU representation (u, ν), and hence the canonical decomposition (p^*, γ^*). First, elicit u using standard methods.[19] By non-triviality and affinity of u, $u(X)$ is convex and contains an interval, so without any loss of generality (rescaling u if necessary), we can assume that $[-1, 1] \subseteq u(X)$, and let $y \in X$ satisfy $u(y) = 0$.

Consider now state $s \in S$. If $\nu(s) \in [0, 1]$, then for any $x \succ y$ we will observe $x \succcurlyeq x\{s\}y \succcurlyeq y$, and therefore, letting $z \sim x\{s\}y$ (which exists by the convexity of $u(X)$), we have the traditional identification $\nu(s) = u(z)/u(x)$ (recall that $u(y) = 0$). If $\nu(s) > 1$, then for an arbitrary $x \succ y$ it follows that $x\{s\}y \succ x \succ y$ and it may *not* be the case that $x\{s\}y$ has a certainty equivalent z. However, there will have to be *some* $x \succ y$ for which there is z such that $z \sim x\{s\}y$,[20] from which it follows that $\nu(s) = u(z)/u(x)$. Finally, if $\nu(s) < 0$, then for arbitrary $x \succ y$ it will be the case that $x\{s\}y \prec y \prec x$, and by a symmetric argument there will have to be some $x \succ y$ and z such that $z \sim x\{s\}y \prec y \prec x$,[21] and thus again $\nu(s) = u(z)/u(x)$.

The elicited ν allows us to compute the canonical decomposition (p^*, γ^*) via the formulas

$$p^*(s) = \frac{|\nu(s)|}{\sum_{t \in S} |\nu(t)|} \qquad \text{and} \qquad \gamma^*(s) = \nu(s) - p^*(s) \quad \text{for all } s \in S.$$

The elicitation procedure introduced above is particularly simple in the special case of SSEU preferences with a unit-bounded representation (see Definition 2). In this case, the WTB in each state lies in the interval $[-1, 1]$, and WTB can be elicited using standard *matching probabilities* (see, e.g., Baillon et al., 2018), even for states with a *negative* WTB. To illustrate the procedure, choose $x \succ y \succ z$ with $y \sim \frac{1}{2}x + \frac{1}{2}z$ and, for each $s \in S$, consider the bet $x\{s\}y$. Under SSEU,

$$V(x\{s\}y) = \nu(s)u(x) + (1 - \nu(s))u(y).$$

Suppose that the SSEU is unit-bounded, that is $|\nu(s)| \leq 1$ for all $s \in S$. In this case, we call the number $\alpha(s) \in [0, 1]$ satisfying

$$\begin{cases} \alpha(s)x + (1 - \alpha(s))y \sim x\{s\}y, & \text{if } x\{s\}y \succcurlyeq y, \\ \alpha(s)z + (1 - \alpha(s))y \sim x\{s\}y, & \text{if } y \succ x\{s\}y, \end{cases} \tag{8}$$

the *extended matching probability* of the bet $x\{s\}y$. If $x\{s\}y \succcurlyeq y$, then (8) yields

$$\alpha(s)u(x) + (1 - \alpha(s))u(y) = \nu(s)u(x) + (1 - \nu(s))u(y),$$

[19] In our Anscombe-Aumann setting, this is immediately done by employing the mixtures of the type $\alpha x + (1-\alpha)y$. In a more general setting, the construction of u can be achieved via standard sequences (see, e.g., Wakker and Deneffe (1996)).

[20] To see this, notice that by construction $u(y) = 0$; thus, $V(x\{s\}y) = \nu(s)u(x)$. We just need to choose x so that $0 < u(x) < (1/\nu(s)) < 1$, and find z so that $u(z) = u(x)\nu(s)$.

[21] In this case, we need to choose x so that $0 < u(x) < (1/(-\nu(s))) < 1$, and then find z so that $-1 < u(z) = u(x)\nu(s) < 0$.

so $\nu(s) = \alpha(s)$. If instead $y \succ x\{s\}y$, then

$$\alpha(s)u(z) + (1 - \alpha(s))u(y) = \nu(s)u(x) + (1 - \nu(s))u(y),$$

and the midpoint condition $y \sim \frac{1}{2}x + \frac{1}{2}z$ implies $\nu(s) = -\alpha(s)$. Equivalently,

$$\nu(s) = \begin{cases} \alpha(s) & \text{if } s \in P, \\ -\alpha(s) & \text{if } s \in N. \end{cases}$$

Thus, the indifferences in (8) allow us to elicit the willingness to bet on a state s by comparing bets with objective lotteries, even for states with a negative WTB.

B Appendix: The Characterization of Unit-bounded SSEU

Given an SSEU preference $\succcurlyeq$ and convex X, choose $x \succ y \succ y' \in X$ satisfying

$$y \sim \frac{1}{2}x + \frac{1}{2}y'. \tag{9}$$

Note that such outcomes always exist by the properties of u and X. They can also be identified in a purely subjective environment via standard sequences (see,e.g., Wakker and Deneffe, 1996).

We can now state the following axiom.

Axiom 10 (Bounded Valence (BV))**.** Given $x \succ y \succ y' \in X$ satisfying Eq. (9), for any $s \in S$, there exists $z \in X$ such that $x \succcurlyeq z \succcurlyeq y'$ and $z \sim x\{s\}y$.

As its name suggests, this axiom places bounds on the effect of valence in the evaluation of the bet $x\{s\}y$. Positive or negative valence may distort the evaluation of this bet relative to its utility consequences alone; in particular, it may happen that $x\{s\}y \succ x$ or that $y \succ x\{s\}y$. The axiom requires, however, that these distortions remain bounded: the certainty equivalent of $x\{s\}y$ must belong to the preference interval $\{z \in X : x \succcurlyeq z \succcurlyeq y'\}$.

Proposition 6. *The binary relation $\succcurlyeq$ satisfies axioms WO, I, A, IS, and BV if and only if it admits a* unit-bounded *SSEU representation (u, ν), i.e., an SSEU representation with $\nu(s) \in [-1, 1]$ for all $s \in S$.*

C Appendix: A Characterization of the Canonical Decomposition

Consider a family of binary relations $\succcurlyeq_E$ which satisfies the assumptions of Proposition 2, including axiom R. In order to identify the canonical probability p^*, we impose an additional

requirement on one mixed information event: there must exist a single mixed event on which conditional preferences are standard SEU and are dynamically consistent—in the sense of iterated conditioning—with the (same-sign) conditionals already disciplined by axiom R. This mixed-event SEU *anchor* provides a behavioral bridge between positive and negative states and allows us to identify (and, with the calibration axiom below, pin down) the tilt.

Axiom 11 (SEU Anchor on a Mixed Event – SA)**.** There exist $s_1^+, s_2^+ \in P_+$ and $s_1^-, s_2^- \in N$ such that, letting $E_0^+ := \{s_1^+, s_2^+\}$, $E_0^- := \{s_1^-, s_2^-\}$, and $E_0 := E_0^+ \cup E_0^-$, the conditional preference $\succcurlyeq_{E_0}$ satisfies axioms WO, I, A, and M. Moreover, for all acts $f, g \in \mathcal{F}$, if $f(s) = g(s)$ for all $s \in E_0$, then $f \sim_{E_0} g$; if $f(s) = g(s)$ for all $s \notin E_0^+$, then $f \succcurlyeq_{E_0} g$ if and only if $f \succcurlyeq_{E_0^+} g$; and if $f(s) = g(s)$ for all $s \notin E_0^-$, then $f \succcurlyeq_{E_0} g$ if and only if $g \succcurlyeq_{E_0^-} f$.

While axiom R does not constrain θ, axiom SA, together with axiom R, makes cross-sign likelihood ratios observable through the mixed event E_0. The next axiom imposes a calibration condition that ties down the remaining degree of freedom using matching probabilities for both the ex-ante preference and the mixed-event conditional preference (as defined in (8)). In the formulation, we say that state s *satisfies the BV property* if, given $x \succ y \succ y'$ satisfying Eq. (9), there exists $z \in X$ such that $x \succcurlyeq z \succcurlyeq y'$ and $z \sim x\{s\}y$. (It follows from our discussion in Appendices A and B that for each such s, matching probabilities are well-defined.)

Axiom 12 (WTB Odds Consistency – WOC)**.** Given the event E_0 from Axiom 11, there exist $s^+ \in \{s_1^+, s_2^+\}$ and $s^- \in \{s_1^-, s_2^-\}$, each satisfying the BV property, such that the matching probabilities $\alpha(s^+)$, $\alpha(s^-)$, $\alpha_{E_0}(s^+)$, and $\alpha_{E_0}(s^-)$ exist uniquely in $(0, 1)$ and satisfy

$$\frac{\alpha_{E_0}(s^+)}{\alpha_{E_0}(s^-)} = \left|\frac{\alpha(s^+)}{\alpha(s^-)}\right|. \tag{10}$$

A matching probability measures how willing the decision maker is, ex-ante, to stake a better outcome on a given state rather than take a sure compromise. Axiom WOC is a *dynamic consistency* requirement for WTB comparisons: it requires that the relative willingness to bet on a representative positive state versus a representative negative state is the same when elicited ex-ante and when elicited again after conditioning on the event E_0. In other words, learning that the state lies in E_0 should not distort the cross-sign betting comparison between s^+ and s^-.

The next result shows that the only valence representation consistent with axioms R, SA, and WOC is the canonical one.

Proposition 7. *The family $(\succcurlyeq_E)_{E \subseteq S}$ satisfies axioms R, SA, and WOC if and only if $\succcurlyeq$ admits the canonical valence representation (u, p^*, γ^*) such that the following hold. First, there exist $s_1^+, s_2^+ \in P_+$ and $s_1^-, s_2^- \in N$ for which, letting*

$$E_0 := \{s_1^+, s_2^+, s_1^-, s_2^-\},$$

the preference $\succcurlyeq_{E_0}$ *is represented by* $(u, p^*(\cdot \mid E_0))$. *Second, for some* $s^+ \in \{s_1^+, s_2^+\}$ *and some* $s^- \in \{s_1^-, s_2^-\}$,

$$p^*(s^+) + \gamma^*(s^+) < 1 \qquad \textit{and} \qquad p^*(s^-) + \gamma^*(s^-) > -1.$$

Third, for every $E \in \mathcal{P} \cup \mathcal{N}$,

$$(u, p^*(\cdot \mid E)) \textit{ represents } \succcurlyeq_E \quad \textit{if } E \in \mathcal{P},$$

and

$$(-u, p^*(\cdot \mid E)) \textit{ represents } \succcurlyeq_E \quad \textit{if } E \in \mathcal{N}.$$

Finally, p^* *is unique in the following sense. If* $p \in \Delta_+(S)$ *is such that* $(u, p, \nu - p)$ *is a valence representation of* $\succcurlyeq$, *and if for every* $E \in \mathcal{P} \cup \mathcal{N}$ *the preference* $\succcurlyeq_E$ *admits the SEU representation* $(u, p(\cdot \mid E))$ *when* $E \in \mathcal{P}$ *and* $(-u, p(\cdot \mid E))$ *when* $E \in \mathcal{N}$, *while* $\succcurlyeq_{E_0}$ *admits the SEU representation* $(u, p(\cdot \mid E_0))$, *then* $p = p^*$.

Therefore, the unique valence representation that (i) updates by Bayesian conditioning on $\mathcal{P} \cup \mathcal{N}$, (ii) admits an SEU anchor on the mixed event E_0, and (iii) is consistent with axioms R and WOC is the canonical decomposition of ν. Equivalently, within the family of dynamically consistent exponential tilts $\mathcal{T}(\nu)$, axiom WOC selects the "center" $\theta = 0$, i.e., the canonical prior p^*.

D Appendix: An Information-theoretic Characterization of $\mathcal{T}(\nu)$

We now give an information-theoretic interpretation of the family $\mathcal{T}(\nu)$. Recall that the canonical probability associated with the signed belief ν is

$$p^* = \frac{|\nu|}{\|\nu\|_v},$$

and assume throughout this section that for ν the set N is nonempty.

Fix $c \in (0, 1)$, and consider the set

$$\mathcal{U}_c(p^*) := \left\{ q \in \Delta_+(S) : \exists \ell : S \to \mathbb{R}_+ \text{ such that } q(s) = \frac{\ell(s)p^*(s)}{\sum_{t \in S} \ell(t)p^*(t)} \ \forall s \in S \text{ and } q(P_+) = c \right\}.$$

Thus, $\mathcal{U}_c(p^*)$ is the set of posterior beliefs that can be obtained from p^* by Bayesian updating after some signal realization which assign posterior probability c to the event P_+.

The next result shows that, among all such posterior beliefs, the family $\mathcal{T}(\nu)$ selects the one that is closest to p^* in relative entropy (as measured by Kullback-Leibler divergence).

Proposition 8. *For every $c \in (0,1)$, the problem*

$$\min_{q \in \mathcal{U}_c(p^*)} \mathrm{D}_{\mathrm{KL}}(q \| p^*)$$

has a unique solution $q_c \in \Delta_+(S)$, which satisfies

$$q_c(s) = \begin{cases} \dfrac{c}{p^*(P)}\, p^*(s), & s \in P, \\ \dfrac{1-c}{p^*(N)}\, p^*(s), & s \in N. \end{cases}$$

Moreover, it holds that

$$\mathcal{T}(\nu) = \{q_c : \ c \in (0,1)\}.$$

Proposition 8 shows that $\mathcal{T}(\nu)$ consists of the posterior beliefs obtained from p^* by the least informative updates that achieve a given posterior mass $q(P) = c$: the signal only provides evidence about whether the true state lies in P or in N, and is otherwise uninformative within each set. To see this, consider the set of all signals ℓ that are constant on P and constant on N. That is, there exist $a, b > 0$ such that

$$\ell(s) = a \quad \text{for all } s \in P, \qquad \ell(s) = b \quad \text{for all } s \in N.$$

Such a signal changes only the relative odds of the two events P and N, and conveys no information about which state occurred within P or within N. Bayes' rule then gives

$$q(s) = \begin{cases} \dfrac{a}{a\, p^*(P) + b\, p^*(N)}\, p^*(s), & s \in P, \\ \dfrac{b}{a\, p^*(P) + b\, p^*(N)}\, p^*(s), & s \in N. \end{cases}$$

From the constraint $q(P) = c$, we obtain

$$q(s) = \begin{cases} \dfrac{c}{p^*(P)}\, p^*(s), & s \in P, \\ \dfrac{1-c}{p^*(N)}\, p^*(s), & s \in N. \end{cases} \tag{11}$$

In particular, the canonical prior p^* is recovered when $c = p^*(P)$, corresponding to a non-informative signal.

E Appendix: Proofs

E.1 Proof of Proposition 1

The first result provides an intermediate step in the proof of Proposition 1.

Proposition 9. *Assume that $\succcurlyeq$ admits the SSEU representation (u, ν). Then there exist $p^+, p^- \in \Delta^+(S)$, and $b \geq 0$ such that*

$$\nu(s) = (1+b)p^+(s) - bp^-(s),$$

for every $s \in S$, so that for every $f \in \mathcal{F}$

$$\int_S u(f)d\nu = (1+b)\int_S u(f)dp^+ - b\int_S u(f)dp^-.$$

Moreover, $\succcurlyeq$ satisfies Axiom 4 if and only if $b = 0$.

Proof. Let $a = \nu^+(S)$ and $b = \nu^-(S)$. Clearly $a \geq 0$, $b \geq 0$, and $\nu(S) = 1$ excludes that $a = b = 0$. If $b = 0$, then $\nu = \nu^+$ and $a = 1$. Moreover, if $a < 1$ then $1 = \nu(S) = a - b$ would imply $b \leq 0$, which contradicts the fact that ν^- is a positive measure. Thus, $a \geq 1$. By setting $p^+(s) = \frac{\nu^+(s)}{a}$ and $p^- = \frac{\nu^-(s)}{b}$ (if $b > 0$, otherwise $p^+ = \nu^+ = \nu$) for every $s \in S$, we have that $p^+, p^- \in \Delta^+(S)$. Moreover, we have

$$\nu(s) = \nu^+(s) - \nu^-(s) = \frac{a\nu^+(s)}{a} - \frac{b\nu^-(s)}{b} = ap^+(s) - bp^-(s)$$

for every $s \in S$. Now let

$$N' = \{s \in S : x\{s\}y \succ y \text{ for some } y \succ x\},$$

and observe that $s \in N'$ if and only if $\nu(s) < 0$. If $\succcurlyeq$ satisfies M, then $N' = \emptyset$, and since $b = \nu^-(S) = \nu^-(S \cap N) = \nu^-(\emptyset) = 0$. Hence $b = 0$ as desired. If $b = 0$, then $\succcurlyeq$ satisfies M. □

Proof of Proposition 1. By defining $\nu(s) = p(s) + \gamma(s)$, a valence representation can be rewritten as $V(f) = \int_S u(f)d(p+\gamma) = \int_S u(f)d\nu$. The set function ν is such that, for any $E, F \subseteq S$ with $E \cap F = \emptyset$, $\nu(E \cup F) = p(E \cup F) + \gamma(E \cup F) = p(E) + p(F) + \gamma(E) + \gamma(F) = \nu(E) + \nu(F)$, where the second equality follows from p being a probability and Property 1 of γ in Definition 3. Lastly, Condition 2 in Definition 3 implies that $\nu(S) = p(S) + \gamma(S) = 1 + 0 = 1$, so that $\nu \in \Delta(S)$.

For the opposite implication, suppose $\succcurlyeq$ has a SSEU representation (u, ν). By Proposition 9, there is $b \geq 0$, and $p^+, p^- \in \Delta^+(S)$ such that $\nu(s) = (1+b)p^+(s) - bp^-(s)$. The valence representation follows by defining $p(s) = p^+(s)$ and $\gamma(s) = b(p^+(s) - p^-(s))$. □

E.2 Proof of Theorem 1

Several steps of this proof are standard, but we report them for completeness. We denote with B the set of functions from S to $\mathbb{R}$, and given $K \subseteq \mathbb{R}$ we denote with $B(K)$ the set of functions from S to K.

Since X is a convex subset of a vector space, it follows that $\mathcal{F}$ is mixture space and because $\succcurlyeq$ satisfies Axioms 1-3, by Theorem 8 in Herstein and Milnor (1953) there exists $V : \mathcal{F} \to \mathbb{R}$ that represents $\succcurlyeq$ such that

$$V(\alpha f + (1-\alpha)g) = \alpha V(f) + (1-\alpha)V(g),$$

for every $f, g \in \mathcal{F}$ and $\alpha \in [0,1]$. Moreover, V is unique up to positive affine transformations. Further, observe that we can find $\bar{x}, \underline{x} \in X$ such that $V(\bar{x}) = -V(\underline{x}) = 1$. To see this, notice that there exist $\bar{x}, \underline{x} \in X$ such that $\bar{x} \succ \underline{x}$. Indeed, if $x \sim x'$ for every $x, x' \in X$, then it would follow by axiom 5 that $f \sim g$ for every $f, g \in \mathcal{F}$, contradicting Axiom 1. Therefore without loss of generality we can take V such that $V(\bar{x}) = 1$ $V(\underline{x}) = -1$.

Define $u : X \to \mathbb{R}$ by $u(x) = V(x)$ for every $x \in X$. Let $K := u(X)$. Then K is convex by the affinity of V and without loss of generality satisfies $[-1,1] \subseteq K$ by the previous paragraph.

Define a function $U : \mathcal{F} \to B(K)$ as follows: for every $f \in \mathcal{F}$ and $s \in S$

$$U(f)(s) := u(f(s)).$$

Then, U is surjective since $K = u(X)$. Furthermore, the affinity of V implies that for all α and $f, g \in \mathcal{F}$

$$U(\alpha f + (1-\alpha)g) = \alpha U(f) + (1-\alpha)U(g).$$

Now, define a functional I on $B(K)$ by

$$I(\phi) = V\left(U^{-1}(\phi)\right),$$

for all $\phi \in B(K)$. Observe that I is well defined since if $f, g \in \mathcal{F}$ satisfy $U(f) = U(g) = \phi$ then we have that $V(f) = V(g)$ by axiom 5. Clearly, it holds that for all $f \in \mathcal{F}$

$$I(U(f)) = I(u \circ f) = V(f).$$

Moreover, $I(x1_S) = x$ for every $x \in K$.

Now let $\phi, \psi \in B(K)$ and let $\alpha \in [0,1]$. Also, let $f, g \in \mathcal{F}$ be such that $U(f) = a$ and $U(g) = b$. Since U is surjective, such f and g exist. Then, we obtain

$$\begin{aligned}
I(\alpha a + (1-\alpha)b) &= V\left(U^{-1}(\alpha a + (1-\alpha)b)\right) \\
&= V\left(U^{-1}(\alpha U(f) + (1-\alpha)U(g))\right) \\
&= V\left(U^{-1}(U(\alpha f + (1-\alpha)g))\right) \\
&= V(\alpha f + (1-\alpha)g) \\
&= \alpha V(f) + (1-\alpha)V(g) \\
&= \alpha I(a) + (1-\alpha)I(b).
\end{aligned}$$

It follows that I is positively homogeneous, i.e. for every $\alpha > 0$ and $\phi \in B(K)$ such that $\alpha\phi \in B(K)$ it holds $I(\alpha\phi) = \alpha I(\phi)$. Moreover, I is additive, that is for every $\phi, \psi \in B(K)$ such that $\phi + \psi \in B(K)$ it holds $I(\phi + \psi) = I(\phi) + I(\psi)$. To see this point, observe that by positive homogeneity we obtain $I(\phi + \psi) = I\left(2\left(\frac{\phi}{2} + \frac{\psi}{2}\right)\right) = 2I\left(\frac{\phi}{2} + \frac{\psi}{2}\right) = 2\left(\frac{I(\phi)}{2} + \frac{I(\psi)}{2}\right) = I(\phi) + I(\psi)$.

We can now extend I to B as follows. For every $\phi \in B$ there exists $\alpha \geq 1$ such that $\psi := \frac{\phi}{\alpha} \in B(K)$. We can therefore define $\bar{I} : B \to \mathbb{R}$ by

$$\bar{I}(\phi) = \alpha I(\psi).$$

Observe that it is now immediate to satisfies $\bar{I}$ satisfies for every $\alpha \in \mathbb{R}$ and $\phi, \psi \in B$

1. $\bar{I}(\phi + \psi) = \bar{I}(\phi) + \bar{I}(\psi)$;
2. $\bar{I}(\alpha\phi) = \alpha\bar{I}(\phi)$.

Now by the Riesz representation theorem (e.g., see Theorem 6.45 in Axler, 1997) there exists a (unique) $\nu : S \to \mathbb{R}$ such that for every $\phi \in B$

$$\bar{I}(\phi) = \int \phi d\nu = \sum_{s \in S} \phi(s)\nu(s).$$

We can therefore conclude that

$$f \succcurlyeq g \iff V(f) \geq V(g) \iff I(u(f)) \geq I(u(g)) \iff \int u(f)d\nu \geq \int u(g)d\nu.$$

Note that we can assume that $\nu(S) = \sum_{s \in S} \nu(s) = 1$. Indeed, it must be that $\sum_{s \in S} \nu(s) = \nu(S) \neq 0$ since $I(1_S) = 1$. Further, if $\sum_{s \in S} \nu(s) = \nu(S) \neq 1$, then letting let $\tilde{\nu}(s) = \frac{\nu(s)}{\nu(S)}$ and $\tilde{u} = \nu(S)u$, so that

$$V(f) = I(u(f)) = \int u(f)\nu = \int \tilde{u}(f)\tilde{\nu},$$

so that the claim is satisfied.

Finally, if there exists $\nu' \in \Delta(S)$ and an affine function $u' : X \to \mathbb{R}$ such that (u', ν') represents $\succcurlyeq$, then because V is unique up to affine transformations there exists $a > 0$ and $b \in \mathbb{R}$ such that

$$\int u'(f)d\nu' = a \int u(f)d\nu + b$$

for every $f \in \mathcal{F}$ from which we obtain

$$u'(x) = au(x) + b,$$

for every $x \in X$. We can therefore conclude that $\nu' = \nu$ as desired.

E.3 Proof of Corollary 1

(i) implies (ii). With Monotonicity, the Anscombe-Aumann theorem implies that the SSEU representation is actually an SEU representation; that is $\nu \in \Delta_+(S)$. It follows that $\nu = p^*$, or equivalently $\gamma^* \equiv 0$.

(ii) implies (iii). Since $\succcurlyeq$ has the valence representation (u, p^*, γ^*) with $\gamma^* \equiv 0$, for any $s \in S$ and $x \succ y$ we have $V(x\,\{s\}\,y) = p^*(s)(u(x) - u(y)) + u(y) \geq u(y)$ so that $x\,\{s\}\,y \succcurlyeq y$. Thus, $S \subseteq P$ and since $P \subseteq S$, we obtain $P = S$.

(iii) implies (i). If $P = S$, then $\nu(s) \geq 0$ for every $s \in S$ and $\nu(S) = \nu(P) = 1$. Hence, $\succcurlyeq$ admits a SEU representation. By standard results, it follows that $\succcurlyeq$ satisfies axiom M.

E.4 Proof of Proposition 2

We first prove two lemmas.

Lemma 1. *For every $\nu \in \Delta(S)$, if $\mathcal{N} \neq \emptyset$,*

$$\{p \in \Delta^+(S) :\ p^*(\cdot \mid E) = p(\cdot \mid E),\ \forall E \in \mathcal{P} \cup \mathcal{N}\}$$
$$= \{p \in \Delta^+(S) :\ \exists k_1, k_2 > 0 \text{ s.t. } k_1 p^*(P) + k_2 p^*(N) = 1,\ \ p = k_1 p^* \text{ on } P,\ \ p = k_2 p^* \text{ on } N\}.$$

Proof. It is immediate to see that

$$\{p \in \Delta^+(S) :\ p^*(\cdot \mid E) = p(\cdot \mid E),\ \forall E \in \mathcal{P} \cup \mathcal{N}\}$$
$$\supseteq \{p \in \Delta^+(S) :\ \exists k_1, k_2 > 0 \text{ s.t. } k_1 p^*(P) + k_2 p^*(N) = 1,\ \ p = k_1 p^* \text{ on } P,\ \ p = k_2 p^* \text{ on } N\}.$$

Now let $q \in \{p \in \Delta^+(S) :\ p^*(\cdot \mid E) = p(\cdot \mid E),\ \forall E \in \mathcal{P} \cup \mathcal{N}\}$. Then $q(s) = \frac{q(P)}{p^*(P)} p^*(s)$ for every $s \in P$ and $q(s) = \frac{q(N)}{p^*(N)} p^*(s)$ for every $s \in N$ (it follows from the assumption $\mathcal{N} \neq \emptyset$ that there is at least one non-null $s \in N$, and that therefore $p^*(N) > 0$). Let $k_1 := \frac{q(P)}{p^*(P)}$ and $k_2 := \frac{q(N)}{p^*(N)}$. Then $k_1 p^*(P) + k_2 p^*(N) = 1$. Hence,

$$q \in \{p \in \Delta^+(S) :\ \exists k_1, k_2 > 0 \text{ s.t. } k_1 p^*(P) + k_2 p^*(N) = 1,\ \ p = k_1 p^* \text{ on } P,\ \ p = k_2 p^* \text{ on } N\},$$

which delivers the desired result. □

Lemma 2. *Given the set $\mathcal{T}(\nu)$ defined in Eq. (2), if $\mathcal{N} \neq \emptyset$,*

$$\mathcal{T}(\nu) = \{p \in \Delta^+(S) :\ \exists k_1, k_2 > 0 \text{ s.t. } k_1 p^*(P) + k_2 p^*(N) = 1,\ \ p = k_1 p^* \text{ on } P,\ \ p = k_2 p^* \text{ on } N\}.$$

Proof. Take $q \in \mathcal{T}(\nu)$ and choose θ so that

$$q(s) = \frac{e^{\theta \mathbf{1}_P(s)}\, p^*(s)}{\sum_{t \in S} e^{\theta \mathbf{1}_P(t)}\, p^*(t)} \quad \forall s \in S.$$

Set

$$Z := e^\theta p^*(P) + p^*(N) > 0, \qquad k_1 := \frac{e^\theta}{Z} > 0, \qquad k_2 := \frac{1}{Z} > 0.$$

Then, for $s \in P$, $q(s) = \dfrac{e^\theta p^*(s)}{Z} = k_1 p^*(s)$, and for $s \in N$, $q(s) = \dfrac{p^*(s)}{Z} = k_2 p^*(s)$. Moreover,

$$k_1 p^*(P) + k_2 p^*(N) = \frac{e^\theta}{Z} p^*(P) + \frac{1}{Z} p^*(N) = \frac{Z}{Z} = 1.$$

Hence,

$$q \in \{p \in \Delta^+(S) :\ \exists k_1, k_2 > 0 \text{ s.t. } k_1 p^*(P) + k_2 p^*(N) = 1,\ \ p = k_1 p^* \text{ on } P,\ \ p = k_2 p^* \text{ on } N\}.$$

Conversely, let $p \in \Delta^+(S)$ satisfy $p = k_1 p^*$ on P and $p = k_2 p^*$ on N for some $k_1, k_2 > 0$ with $k_1 p^*(P) + k_2 p^*(N) = 1$. Define $\theta := \ln(k_1/k_2)$ and observe that

$$e^\theta p^*(P) + p^*(N) = \frac{k_1}{k_2} p^*(P) + p^*(N) = \frac{1}{k_2} =: Z.$$

Then, for $s \in P$,

$$\frac{e^{\theta} p^*(s)}{Z} = \frac{k_1}{k_2} \cdot k_2\, p^*(s) = k_1 p^*(s) = p(s),$$

and for $s \in N$,

$$\frac{p^*(s)}{Z} = k_2 p^*(s) = p(s).$$

Hence, we can conclude that

$$p(s) = \frac{e^{\theta\, \mathbf{1}_P(s)}\, p^*(s)}{\sum_{t \in S} e^{\theta\, \mathbf{1}_P(t)}\, p^*(t)} \quad \forall s \in S.$$

which implies that $p \in \mathcal{T}(\nu)$. □

Proof of Proposition 2. It is routine to prove that axiom R is implied by the representation. Conversely, assume that the family $(\succcurlyeq_E)_{E \subseteq S}$ satisfies axiom R. If $\mathcal{N} = \emptyset$, $\mathcal{T}(\nu) = p^*$ and the result holds trivially, so henceforth assume $\mathcal{N} \neq \emptyset$. When $E \in \mathcal{P}$, by axiom R we obtain

$$\begin{aligned} f \succcurlyeq_E g &\iff fEh \succcurlyeq_E gEh \iff fEh \succcurlyeq gEh \\ &\iff \int_E u \circ f\, d\nu + \int_{E^c} u \circ h\, d\nu \geq \int_E u \circ g\, d\nu + \int_{E^c} u \circ h\, d\nu \\ &\iff \int_E u \circ f\, d\nu \geq \int_E u \circ g\, d\nu \\ &\iff \int u \circ f\, dp^*(\cdot|E) \geq \int u \circ g\, dp^*(\cdot|E). \end{aligned}$$

This shows that $\succcurlyeq_E$ has a SEU representation $(u, p^*(\cdot|E))$. When $E \in \mathcal{N}$, again by axiom R we obtain

$$\begin{aligned} f \succcurlyeq_E g &\iff fEh \succcurlyeq_E gEh \iff fEh \succcurlyeq gEh \\ &\iff \int_E u \circ f\, d\nu + \int_{E^c} u \circ h\, d\nu \geq \int_E u \circ g\, d\nu + \int_{E^c} u \circ h\, d\nu \\ &\iff \int_E u \circ f\, d\nu \geq \int_E u \circ g\, d\nu \\ &\iff \int -u \circ f\, dp^*(\cdot|E) \geq \int -u \circ g\, dp^*(\cdot|E). \end{aligned}$$

This shows that $\succcurlyeq_E$ has a SEU representation $(-u, p^*(\cdot|E))$.

Finally, assume that $p \in \Delta^+(S)$ is such that $(u, p, \nu - p)$ is a valence representation of $\succcurlyeq$ and, for every $E \in \mathcal{P} \cup \mathcal{N}$, $\succcurlyeq_E$ admits the SEU representation $(u, p(\cdot \mid E))$ when $E \in \mathcal{P}$ and $(-u, p(\cdot \mid E))$ when $E \in \mathcal{N}$. Then, by Lemma 1 we conclude that

$$p \in \{q \in \Delta^+(S) :\ \exists k_1, k_2 > 0 \text{ s.t. } k_1 p^*(P) + k_2 p^*(N) = 1,\ \ q = k_1 p^* \text{ on } P,\ \ q = k_2 p^* \text{ on } N\}.$$

Lemma 2 then implies that $p \in \mathcal{T}(\nu)$, as desired. □

E.5 Proof of Proposition 3

(i) ⇒ (ii). Fix s and set x such that $u(x) = \nu_2(s)$ so that $1\{s\}0 \sim_2 x$. If $x < 0$, then $1\{s\}0 \preccurlyeq_1 x$, i.e. $\nu_1(s) \leq x < 0$, whence $s \in N_1$. As every $s \in N_2$ has $\nu_2(s) < 0$, we get $N_2 \subseteq N_1$.

Now if $x < 0$, then $\nu_1(s) \le x < 0$, so $|\nu_1(s)| = -\nu_1(s) \ge -x = |x| = |\nu_2(s)|$. If $x \ge 0$, then either $\nu_1(s) \ge x$ or $\nu_1(s) \le -x$, and in either case $|\nu_1(s)| \ge x = |\nu_2(s)|$. Thus $|\nu_1(s)| \ge |\nu_2(s)|$ for all s.

(ii) $\Rightarrow$ (i). Fix s and set $x \in X$ such that $u(x) = \nu_2(s)$ with $1\{s\}0 \sim_2 x$. If $x < 0$, then $s \in N_2 \subseteq N_1$, so $\nu_1(s) \le 0$. From $|\nu_1(s)| \ge |x| = -x$ and $\nu_1(s) \le 0$ we get $-\nu_1(s) \ge -x$, i.e. $\nu_1(s) \le x$, hence $1\{s\}0 \preccurlyeq_1 x$. If $x \ge 0$, then $|\nu_1(s)| \ge x$ implies either $\nu_1(s) \ge x$ (so $1\{s\}0 \succcurlyeq_1 x$) or $\nu_1(s) \le -x$ (so $1\{s\}0 \preccurlyeq_1 (-x)$), establishing *(i)* as desired.

E.6 Calculations for Example 7

Note that we have $\gamma_c^* := \nu_c - p^*$, so that

$$\gamma_c^*(s) = \begin{cases} |\nu(s)|\left(c - \dfrac{1}{\|\nu\|_v}\right), & s \in P \setminus F, \\ -|\nu(s)|\left(c + \dfrac{1}{\|\nu\|_v}\right), & s \in F \cup N. \end{cases}$$

Since

$$|\nu(s) - p^*(s)| = \begin{cases} |\nu(s)|\left(1 - \dfrac{1}{\|\nu\|_v}\right), & s \in P, \\ |\nu(s)|\left(1 + \dfrac{1}{\|\nu\|_v}\right), & s \in N, \end{cases}$$

we have, for every s with $|\nu(s)| > 0$,

$$|\gamma_c^*(s)| = \begin{cases} |\nu(s)|\left(c - \dfrac{1}{\|\nu\|_v}\right) \ge |\nu(s)|\left(1 - \dfrac{1}{\|\nu\|_v}\right) = |\gamma^*(s)|, & s \in P \setminus F, \\ |\nu(s)|\left(c + \dfrac{1}{\|\nu\|_v}\right) > |\nu(s)|\left(1 - \dfrac{1}{\|\nu\|_v}\right) = |\gamma^*(s)|, & s \in F, \\ |\nu(s)|\left(c + \dfrac{1}{\|\nu\|_v}\right) \ge |\nu(s)|\left(1 + \dfrac{1}{\|\nu\|_v}\right) = |\gamma^*(s)|, & s \in N. \end{cases}$$

Equality in the first and third lines occurs only when $c = 1$. For $s \in F$, the inequality is strict. At states with $|\nu(s)| = 0$, both sides are equal to zero. Hence

$$|\gamma_c^*(s)| \ \ge \ |\gamma^*(s)| \qquad \text{for every } s \in S,$$

and the inequality is strict at every s with $|\nu(s)| > 0$ whenever $c > 1$.

E.7 Proof of Proposition 4

Lemma 3. *Let $x = (x_i)_{i=1}^n, y = (y_i)_{i=1}^n \in \{z \in \mathbb{R}^n : \sum_{i=1}^n z_i = 1\}$ be such that for some $c > 0$ it holds $|x_i| = c|y_i|$ for every $i = 1, \dots, n$, $y_j < 0$ for some $j = 1, \dots, n$, and $\{i \in \{1, \dots, n\} : y_i < 0\} \subseteq \{i \in \{1, \dots, n\} : x_i < 0\}$. Then $c \ge 1$ if and only if for every $i = 1, \dots, n$*

$$\left| x_i - \frac{|x_i|}{\|x\|_v} \right| \ge \left| |y_i| - \frac{|y_i|}{\|y\|_v} \right|.$$

Proof. From $|x_i| = c\,|y_i|$ for all i, we have

$$\|x\|_v = \sum_i |x_i| = \sum_i c\,|y_i| = c\,\|y\|_v, \qquad \text{and hence} \qquad \frac{|x_i|}{\|x\|_v} = \frac{|y_i|}{\|y\|_v} \ \ (\forall i).$$

Therefore, if we set $\alpha_i := \frac{|y_i|}{\|y\|_v}$, the desired family of inequalities is equivalent to

$$|x_i - \alpha_i| \ \geq \ |y_i - \alpha_i| \qquad (\forall i).$$

Since $y_j < 0$ for some j and $\sum_i y_i = 1$, we have $\|y\|_v > 1$, hence $\delta := \frac{1}{\|y\|_v} \in (0,1)$.

Now to see why $c \geq 1$ is necessary, pick j with $y_j < 0$. By the inclusion $\{y_i < 0\} \subseteq \{x_i < 0\}$, we also have $x_j < 0$, and from $|x_j| = c\,|y_j|$ it follows that $x_j = c\,y_j$. Then

$$|y_j - \alpha_j| = |y_j - \tfrac{|y_j|}{\|y\|_v}| = |y_j - (-y_j)\delta| = |y_j|\,(1+\delta),$$

and

$$|x_j - \alpha_j| = |c\,y_j - \tfrac{|y_j|}{\|y\|_v}| = |c\,y_j - (-y_j)\delta| = |y_j|\,(c+\delta).$$

Thus $|x_j - \alpha_j| \geq |y_j - \alpha_j|$ is equivalent to $c + \delta \geq 1 + \delta$, i.e. $c \geq 1$.

Conversely, assume $c \geq 1$ and consider $i \in \{1, \ldots, n\}$. If $y_i < 0$, it follows that $x_i < 0$ and hence $x_i = c\,y_i$, and the same computation as above yields $|x_i - \alpha_i| = |y_i|\,(c+\delta) \geq |y_i|\,(1+\delta) = |y_i - \alpha_i|$.

If $y_i = 0$, then $|x_i| = c\,|y_i| = 0$ so $x_i = 0$, and both sides are 0.

Finally, if $y_i > 0$, then either $x_i = c\,y_i$ or $x_i = -c\,y_i$. If $x_i = c\,y_i$, then

$$|x_i - \alpha_i| = |c\,y_i - \tfrac{y_i}{\|y\|_v}| = y_i\,|c - \delta| \ \geq \ y_i\,(1-\delta) = |y_i - \alpha_i|,$$

because $c \geq 1$ implies $c - \delta \geq 1 - \delta$. If $x_i = -c\,y_i$, then

$$|x_i - \alpha_i| = |\,-c\,y_i - \tfrac{y_i}{\|y\|_v}| = y_i\,(c+\delta) \ \geq \ y_i\,(1-\delta) = |y_i - \alpha_i|,$$

since $c \geq 1$ and $\delta \geq 0$. Hence, in all cases we have that $|x_i - \alpha_i| \geq |y_i - \alpha_i|$. □

Lemma 4. *Let $\succcurlyeq_1$ and $\succcurlyeq_2$ admit SSEU representations (u, ν_1) and (u, ν_2) with $u(x) = x$. Then 1 is uniformly more non-classical than 2 if and only if there exists $c \geq 1$ such that for every $s \in S$*

$$\nu_1(s) \ = \ \begin{cases} c\,\nu_2(s) & \text{if } \nu_2(s) < 0, \\ \pm\, c\,\nu_2(s) & \text{if } \nu_2(s) \geq 0. \end{cases} \tag{12}$$

Proof. Suppose that there exists $\alpha \in (0,1]$ such that for every state s, if $1_{\{s\}}0 \sim_2 \alpha x$ with $x < 0$, then $1_{\{s\}}0 \sim_1 x$, and if $x \geq 0$ then $1_{\{s\}}0 \sim_1 x$ or $1_{\{s\}}0 \sim_1 -x$. Observe that since $u(x) = x$, we obtain $\nu_2(s) = \alpha u(x)$. It follows that if $x < 0$, then $\nu_1(s) = u(x)$. If $x \geq 0$, then $\nu_1(s) = u(x)$ or $\nu_1(s) = -u(x)$. Letting $c := \frac{1}{\alpha} \geq 1$, we therefore obtain that

$$\nu_1(s) \ = \ \begin{cases} c\,\nu_2(s) & \text{if } \nu_2(s) < 0, \\ \pm\, c\,\nu_2(s) & \text{if } \nu_2(s) \geq 0, \end{cases}$$

as desired.

Conversely, suppose there exists $c \geq 1$ such that (12) holds. Let $\alpha := \frac{1}{c} \in (0,1]$. Assume that x is such that $1_{\{s\}}0 \sim_2 \alpha x$. It follows that $\nu_2(s) = \frac{1}{c}u(x)$. Then if $x < 0$, using (12), we obtain that $\nu_1(s) = u(x) = x$ and so $1_{\{s\}}0 \sim_1 x$. If $x \geq 0$, then from (12) we conclude that either $\nu_1(s) = u(x) = x$ or $\nu_1(s) = -u(x) = -x$. Therefore, either $1_{\{s\}}0 \sim_1 x$ or $1_{\{s\}}0 \sim_1 -x$. Hence, 1 is uniformly more non-classical than 2, as desired. □

Proof of Proposition 4. $(i) \implies (ii)$. Since DM 1 is uniformly more non-classical than 2, Lemma 4 implies that $N_2 \subseteq N_1$ and there exists $c \geq 1$ such that $|\nu_1(s)| = c\,|\nu_2(s)|$. It follows that $p_1^* = p_2^*$. If $N_2 = \emptyset$, then we are done. If $N_2 \neq \emptyset$, then by applying Lemma 3 we obtain that for all $s \in S$

$$|\gamma_1^*(s)| = \left|\nu_1(s) - \frac{|\nu_1(s)|}{\sum_{s' \in S}|\nu_1(s')|}\right| \geq \left|\nu_2(s) - \frac{|\nu_2(s)|}{\sum_{s' \in S}|\nu_2(s')|}\right| = |\gamma_2^*(s)|,$$

as desired.

$(ii) \implies (i)$. Since $p_1^* = p_2^*$, letting

$$c = \frac{\sum_{s' \in S}|\nu_1(s')|}{\sum_{s' \in S}|\nu_2(s')|},$$

we obtain that $|\nu_1(s)| = c|\nu_2(s)|$ for every $s \in S$. If $N_2 = \emptyset$, then $c \geq 1$. Thus applying Lemma 4 we obtain that DM 1 is uniformly more non-classical than DM 2. If $N_2 \neq \emptyset$, then applying Lemma 3 we obtain that $c \geq 1$. Hence, by Lemma 4 we obtain that DM 1 is uniformly more non-classical than DM 2. □

E.8 Proof of Fact 3

According to SSEU, the difference between $V(f_2)$ and $V(f_1)$ is

$$u(x-\psi) - \nu(L)u(x-\ell) - (1-\nu(L))u(x).$$

Since $0 \leq \nu(L) \leq 1$, Jensen's inequality yields

$$V(f_2) - V(f_1) = u(x-\psi) - \nu(L)u(x-\ell) - (1-\nu(L))u(x) \geq$$
$$u(x-\psi) - u(x-\nu(L)\ell) = 0.$$

Thus, the DM buys the insurance policy. The SSEU difference between f_4 and f_2 is

$$V(f_4) - V(f_2) = \nu(E)u(x-\psi-\pi+w) + (1-\nu(E))u(x-\psi-\pi) - u(x-\psi).$$

By assumption, $\nu(E) = p(E)+\gamma(E) = 1+\epsilon$ for some $\epsilon \geq 0$. Also, $u(x-\psi-\pi+w) - u(x-\psi) \geq 0$ since u is increasing and $w \geq \pi$. Similarly, $u(x-\psi-\pi+w) - u(x-\psi-\pi) \geq 0$. Therefore

$$u(x-\psi-\pi+w) + \epsilon(u(x-\psi-\pi+w) - u(x-\psi-\pi)) - u(x-\psi) \geq 0,$$

because $w \geq \pi$, and thus $V(f_4) \succcurlyeq V(f_2)$. We conclude that $V(f_4) \succcurlyeq V(f_2) \succcurlyeq V(f_1)$.

E.9 Proof of Fact 4

If $p(E) + \gamma(E) \geq 1$, then $\gamma(E) \geq 1 - p(E)$. Since $u(x) \geq u(y)$, we have $\gamma(E)[u(x) - u(y)] \geq (1 - p(E))[u(x) - u(y)]$. Adding and subtracting $p(E)u(x)$ on the right-hand side implies

$$\gamma(E)[u(x) - u(y)] \geq u(x) - [p(E)u(x) + (1 - p(E))u(y)].$$

Since u is increasing, we can write $u(x) \geq u(p(E)x + (1 - p(E))y)$, and thus

$$\gamma(E)[u(x) - u(y)] \geq u(p(E)x + (1 - p(E))y) - [p(E)u(x) + (1 - p(E))u(y)],$$

as required.

E.10 Proof of Theorem 2

Necessity of the axioms is straightforward. As to sufficiency, since X is mixture set and since $\succcurlyeq$ satisfies axioms WO, C and RI, by Theorem 8 in Herstein and Milnor (1953) there exists $u : X \to \mathbb{R}$ that represents the restriction of $\succcurlyeq$ on X and such that

$$u\left(\alpha x + (1 - \alpha)y\right) = \alpha u(x) + (1 - \alpha)u(y),$$

for every $x, y \in X$ and $\alpha \in [0, 1]$. Moreover, u is unique up to positive affine transformations. Axiom S implies that each act f has a certainty equivalent $x_f \in X$.

Now, define a functional $I : B_0(2^S, \text{Range}(u)) \to \mathbb{R}$ as $I(a) = u(x_f)$ where $u(f)(s) = a(s)$. To see that I is well-defined, suppose that $u(f(s)) = a(s) = b(s) = u(g(s))$ for all $s \in S$. Then by IS $f \sim g$, therefore $I(a) = u(x_f) = u(x_g) = I(b)$, so I is well-defined. By construction I is normalized and constant-archimedean, and it can be proved using standard arguments that it is also continuous (w.r.t. to the sup norm).

Remark 2. *As mentioned in the main text, Solvability may impose restrictions on the way in which our DM can assign valence to events when assessing bets with very high or very low payoffs. This will be the case if the utility u has a bounded range. The only (non-SEU) case in which Solvability imposes no restrictions is when Range$(u) = \mathbb{R}$, which describes exactly the intersection of the BC model and the SSEU model.*[22]

E.11 Proof of Proposition 5

Suppose that $\succcurlyeq$ has a BC representation. Let define, for $x, y \in X$ with $x \succ y$, the set function $\rho_{x,y} : 2^S \to \mathbb{R}$ as $\rho_{x,y}(E) = \frac{I(u(xEy)) - u(y)}{u(x) - u(y)}$. By definition, $\rho_{x,y}(S) = 1$ and $\rho_{x,y}(\emptyset) = 1 - \rho_{x,y}(S) = 0$,

[22] Given an SSEU preference (u, ν) with Range(u) bounded, suppose that $E \subseteq S$ is such that $\nu(E) = 1 + \alpha$ for some $\alpha > 0$ and choose a normalization of u such that $u(x_0) = 0$ for some $x_0 \in X$. Then if $\sup(\text{Range}(u)) = M$, there is going to be a $u(z) = M - \epsilon$ such that the bet zEx_0 has $V(zEx_0) = (M - \epsilon)(1 + \alpha) > M$, so that it does not have a certainty equivalent.

thus ρ is a signed capacity. By Cerreia-Vioglio et al. (2012, Proposition 7), any signed capacity of bounded variation $\rho_{x,y}$ can be decomposed into $\rho_{x,y} = \rho^+_{x,y} - \rho^-_{x,y}$, where $\rho^+_{x,y}$ and $\rho^-_{x,y}$ are capacities. Our signed capacity $\rho_{x,y}$ is clearly of bounded variation since the state space is finite. Thus, we can write $\rho_{x,y} = a_{x,y}\mu^+_{x,y} - b_{x,y}\mu^-_{x,y}$, where $a_{x,y} = \rho^+_{x,y}(S)$, $\mu^+_{x,y} = \rho^+_{x,y}/a_{x,y}$, $b_{x,y} = \rho^-_{x,y}(S)$, and $\mu^-_{x,y} = \rho^-_{x,y}/b_{x,y}$. Normalization excludes $a_{x,y} = b_{x,y} = 0$. If $b_{x,y} = 0$, then we define $\mu^+_{x,y} = \rho_{x,y}$, $a_{x,y} = 1$ and the result follows. With these definitions, we get $\mu^+_{x,y}(S) = \mu^-_{x,y}(S) = 1$, and the normalization $\rho_{x,y}(S) = 1$ implies $a_{x,y} - b_{x,y} = 1$. To obtain the decomposition above, we then set $\Gamma_{x,y}(E) = b_{x,y}(\mu^+_{x,y}(E) - \mu^-_{x,y}(E))$ for any $E \subseteq S$. Thus, for all $x \succ y$ in X and all $E \subseteq S$, $I(u(xEy)) = \rho_{x,y}(E)u(x) + (1 - \rho_{x,y}(E))u(y)$ or, equivalently, $I(u(xEy)) = (\mu^+_{x,y}(E) + \Gamma_{x,y}(E))u(x) + (1 - \mu^+_{x,y}(E) - \Gamma_{x,y}(E))u(y)$ yielding the generalized valence representation.

E.12 Proof of Proposition 6

Suppose first that (u, ν) is a unit-bounded SSEU representation. By Theorem 1, axioms WO, I, A, and IS are satisfied. To show that BV is satisfied, consider $x, y, y' \in X$ satisfying Eq. (9). Then

$$u(y) - u(y') = u(x) - u(y),$$

so y', y, x are equally spaced in utility.

Fix $s \in S$. If $\nu(s) \in [0, 1]$, then

$$V(x\{s\}y) = u(y) + \nu(s)\Big(u(x) - u(y)\Big) \in [u(y), u(x)].$$

Since $u(X)$ is convex, there exists $z \in [y, x]$[23] such that

$$u(z) = V(x\{s\}y),$$

hence $z \sim x\{s\}y$.

If instead $\nu(s) \in [-1, 0)$, then

$$V(x\{s\}y) = u(y) + \nu(s)\Big(u(x) - u(y)\Big) \in [u(y'), u(y)),$$

because $u(y') = u(y) - (u(x) - u(y))$. Again by convexity of $u(X)$, there exists $z \in [y', y)$ such that

$$u(z) = V(x\{s\}y),$$

and hence $z \sim x\{s\}y$.

Therefore, in all cases there exists $z \in [y', x]$ such that $z \sim x\{s\}y$, so BV holds.

[23] With a slight abuse of notation, for any $x \succ y \in X$ we henceforth denote $[y, x] = \{z \in X : x \succcurlyeq z \succcurlyeq y\}$ (and analogous for strict preferences).

Conversely, suppose that $\succcurlyeq$ satisfies WO, I, A, IS, and BV. By Theorem 1, it admits an SSEU representation (u, ν). Let $x, y, y' \in X$ satisfy Eq. (9). As above,

$$u(y) - u(y') = u(x) - u(y).$$

By BV, for every $s \in S$ there exists $z \in [y', x]$ such that

$$z \sim x\{s\}y.$$

Hence

$$u(z) = V(x\{s\}y) = u(y) + \nu(s)\Big(u(x) - u(y)\Big),$$

so

$$\nu(s) = \frac{u(z) - u(y)}{u(x) - u(y)}.$$

Now normalize so that $u(y) = 0$ and $u(x) = 1$. Using the equal-spacing condition we obtain $u(y') = -1$, and therefore $u(z) \in [-1, 1]$. Thus

$$\nu(s) = u(z) \in [-1, 1].$$

Since s was arbitrary, the representation is unit-bounded.

E.13 Proof of Proposition 7

The proof that the axioms are sufficient for the representation and its uniqueness is divided in several steps.

Step 1: Canonical valence representation. By Axiom R and Proposition 2, the ex-ante preference $\succcurlyeq$ admits an SSEU representation (u, ν) and a canonical valence representation (u, p^*, γ^*). Moreover, for every $E \in \mathcal{P} \cup \mathcal{N}$, $\succcurlyeq_E$ is SEU with $(u, p^*(\cdot \mid E))$ when $E \subseteq P$ and with $(-u, p^*(\cdot \mid E))$ when $E \subseteq N$. This establishes the second requirement in the statement.

Step 2: Unit-boundedness at two states. By Axiom WOC there exist $s^+ \in P_+$ and $s^- \in N$ such that the (extended) matching probabilities $\alpha(s^+)$ and $\alpha(s^-)$ exist uniquely and belong to $(0, 1)$. By (8), this implies $|\nu(s^+)| < 1$ and $|\nu(s^-)| < 1$. Since $s^+ \in P_+$ and $s^- \in N$, we have $\nu(s^+) > 0$ and $\nu(s^-) < 0$, hence

$$\nu(s^+) < 1 \qquad \text{and} \qquad \nu(s^-) > -1.$$

Using $\nu = p^* + \gamma^*$, this is equivalent to

$$p^*(s^+) + \gamma^*(s^+) < 1 \qquad \text{and} \qquad p^*(s^-) + \gamma^*(s^-) > -1,$$

which is the first requirement in the statement.

Step 3: Using the mixed-event anchor to identify beliefs. Let $E_0 = \{s_1^+, s_2^+, s_1^-, s_2^-\}$ be as in Axiom SA, and write $E_0^+ = \{s_1^+, s_2^+\}$ and $E_0^- = \{s_1^-, s_2^-\}$. By Axiom SA, $\succcurlyeq_{E_0}$ admits a SEU representation. Let (u, π) be a SEU representation of $\succcurlyeq_{E_0}$.

By the dynamic consistency part in Axiom SA, on E_0^+ the conditional probability induced by π must agree with the probability representing $\succcurlyeq_{E_0^+}$, and on E_0^- the conditional probability induced by π must agree with the probability representing $\succcurlyeq_{E_0^-}$.

On the other hand, $E_0^+ \subseteq P$ and $E_0^- \subseteq N$, so by Step 1 (Axiom R), $\succcurlyeq_{E_0^+}$ is represented by $(u, p^*(\cdot \mid E_0^+))$ and $\succcurlyeq_{E_0^-}$ is represented by $(-u, p^*(\cdot \mid E_0^-))$. Therefore,

$$\pi(\cdot \mid E_0^+) = p^*(\cdot \mid E_0^+) \qquad \text{and} \qquad \pi(\cdot \mid E_0^-) = p^*(\cdot \mid E_0^-).$$

Equivalently, there exist constants $c_+, c_- > 0$ such that

$$\pi(s) = c_+ \, p^*(s) \text{ for } s \in E_0^+, \qquad \pi(s) = c_- \, p^*(s) \text{ for } s \in E_0^-.$$

Step 4: WOC pins down c^+/c^-. Fix $s^+ \in \{s_1^+, s_2^+\}$ and $s^- \in \{s_1^-, s_2^-\}$ as in Axiom WOC. Because $\succcurlyeq_{E_0}$ is SEU with representation (u, π), the matching probabilities are well-defined and unique, and they satisfy

$$\frac{\alpha_{E_0}(s^+)}{\alpha_{E_0}(s^-)} = \frac{\pi(s^+)}{\pi(s^-)} = \frac{c_+}{c_-} \frac{p^*(s^+)}{p^*(s^-)}.$$

By Axiom WOC,

$$\frac{\alpha_{E_0}(s^+)}{\alpha_{E_0}(s^-)} = \left| \frac{\alpha(s^+)}{\alpha(s^-)} \right|.$$

By the definition of p^*,

$$\left| \frac{\alpha(s^+)}{\alpha(s^-)} \right| = \frac{|\nu(s^+)|}{|\nu(s^-)|} = \frac{p^*(s^+)}{p^*(s^-)}.$$

Combining the previous equations we obtain that $c_+ = c_-$. It follows that

$$\pi(\cdot) = p^*(\cdot \mid E_0).$$

Therefore, $\succcurlyeq_{E_0}$ is represented by $(u, p^*(\cdot \mid E_0))$, as claimed.

Step 5: Uniqueness. For the final statement, let $p \in \Delta_+(S)$ be such that $(u, p, \nu - p)$ is a valence representation of $\succcurlyeq$, and suppose that for every $E \in \mathcal{P} \cup \mathcal{N}$ the conditional preference $\succcurlyeq_E$ is represented by $(u, p(\cdot \mid E))$ when $E \subseteq P$ and by $(-u, p(\cdot \mid E))$ when $E \subseteq N$, while $\succcurlyeq_{E_0}$ is represented by $(u, p(\cdot \mid E_0))$.

By Proposition 2, necessarily $p \in T(\nu)$, so there exists $\theta \in \mathbb{R}$ such that

$$\frac{p(s^+)}{p(s^-)} = e^\theta \frac{p^*(s^+)}{p^*(s^-)} \qquad \text{for all } s^+ \in P, \ s^- \in N.$$

Since $\succcurlyeq_{E_0}$ is SEU with $(u, p(\cdot \mid E_0))$, we have $\alpha_{E_0}(s) = p(s \mid E_0)$ for $s \in E_0$, and hence

$$\frac{\alpha_{E_0}(s^+)}{\alpha_{E_0}(s^-)} = \frac{p(s^+ \mid E_0)}{p(s^- \mid E_0)} = \frac{p(s^+)}{p(s^-)}.$$

Axiom WOC and the preservation of absolute WTB ratios by p^* yield

$$\frac{p(s^+)}{p(s^-)} = \frac{\alpha_{E_0}(s^+)}{\alpha_{E_0}(s^-)} = \left| \frac{\alpha(s^+)}{\alpha(s^-)} \right| = \frac{p^*(s^+)}{p^*(s^-)}.$$

Combining with the tilt identity gives $e^\theta = 1$, hence $\theta = 0$ and therefore $p = p^*$.

It is routine to prove the converse statement, and therefore we omit the proof.

E.14 Proof of Proposition 8

Let $q \in \mathcal{U}_c(p^*)$. First, observe that $\mathrm{D_{KL}}\big(q \| p^*\big) < \infty$. Moreover, since $q(P) = c$ and $q(N) = 1 - c$, so we obtain

$$\begin{aligned}\mathrm{D_{KL}}(q\|p^*) &= \sum_{s\in S} q(s) \log \frac{q(s)}{p^*(s)} \\ &= c\ \mathrm{D_{KL}}\Big(q(\cdot \mid P)\,\|\,p^*(\cdot \mid P)\Big) + (1-c)\ \mathrm{D_{KL}}\Big(q(\cdot \mid N)\,\|\,p^*(\cdot \mid N)\Big) \\ &\quad + c \log \frac{c}{p^*(P)} + (1-c) \log \frac{1-c}{p^*(N)}.\end{aligned}$$

The last two terms depend only on c, hence are constant on $\mathcal{U}_c(p^*)$. The first two terms are nonnegative, and each vanishes if and only if the corresponding conditional distributions coincide. Therefore, the unique minimizer is the posterior q_c such that

$$q_c(\cdot \mid P) = p^*(\cdot \mid P) \qquad \text{and} \qquad q_c(\cdot \mid N) = p^*(\cdot \mid N).$$

Since $q_c(P) = c$ and $q_c(N) = 1 - c$, this yields

$$q_c(s) = \begin{cases} \dfrac{c}{p^*(P)}\, p^*(s), & s \in P, \\ \dfrac{1-c}{p^*(N)}\, p^*(s), & s \in N. \end{cases} \tag{13}$$

Now let

$$\theta(c) := \log \frac{c}{1-c} - \log \frac{p^*(P)}{p^*(N)}.$$

Then

$$e^{\theta(c)} = \frac{c\, p^*(N)}{(1-c)\, p^*(P)}.$$

If we set

$$Z(c) := \sum_{t\in S} e^{\theta(c)\mathbf{1}_P(t)}\, p^*(t) = e^{\theta(c)} p^*(P) + p^*(N),$$

then a direct computation gives

$$\frac{e^{\theta(c)}}{Z(c)} = \frac{c}{p^*(P)} \qquad \text{and} \qquad \frac{1}{Z(c)} = \frac{1-c}{p^*(N)}.$$

Substituting into (13) yields

$$q_c(s) = \frac{e^{\theta(c)\mathbf{1}_P(s)}}{\sum_{t\in S} p^*(t) e^{\theta(c)\mathbf{1}_P(t)}}\, p^*(s)$$

and therefore

$$\mathcal{T}(\nu) = \{q_c :\ c \in (0,1)\}.$$

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